# ON NOTE-TAKING

# SOBRE ANOTAÇÕES

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## ABSTRACT

Note-taking, or copying quickly and accurately the material that the professor puts on the blackboard, is the predominate – and preferred – student activity in college-level mathematics classrooms. This activity is herein investigated in relation to the following eleven topics in the constructivist theory of mathematics education: student-centered environment, the professor as an authority figure, dialogue and participant activities, personal autonomy, self-reliance, real math, interconnected cognitive spaces, metacognition, assessment, interpersonal relations and social values. In each case, it is found that the activity of taking notes is not compatible with the constructivist position on these topics. A few remarks are also made regarding note-taking in more general, non-constructivist settings.
**Keywords**: Theories of Mathematics Education; Constructivism; Note-taking.

## RESUMO

Fazer apontamentos, ou seja, copiar rapidamente e com precisão o material que o professor escreve no quadro-negro, é a atividade predominante – e esperada – dos alunos na sala de aula de matemática ao nível universitário. No presente trabalho, investiga-se a relação da referida atividade com os seguintes onze tópicos da teoria construtivista de Educação Matemática: aulas centradas no aluno, o professor como figura de autoridade, diálogo e atividades participantes, autonomia, autoconfiança, matemática verdadeira, espaços cognitivos interligados, metacognição, avaliação, relações interpessoais e valores sociais. Conclui-se, em cada caso, que a atividade de fazer apontamentos não é compatível com a posição construtivista sobres esses tópicos. Faz-se ainda alguns comentários sobre a atividade de fazer apontamentos em contextos não-construtivistas.
**Palavras-chave**: Teorias de Educação Matemática; Construtivismo; Fazendo Apontamentos.

According to a current description,

> Were we to be transported to a "typical" mathematics classroom this very moment, we would in all likelihood find the teacher mechanically solving a "problem" – or, rather, copying out the solution from the text – while mumbling some gibberish, apparently about the problem, to the blackboard. The teacher's dulled charges, long since accustomed to proper classroom etiquette, sit quietly awaiting recess. Unfortunately, no mathematics is being done in this classroom. (FOSSA, 2019, p. 10.)

The description is clearly meant to depict the primary (and, perchance, secondary mathematics classroom); nevertheless, were we to shuttle over to a college class, not much

would be different. Our little masters would indeed be replaced by hulking youths, but they too would be just as dulled, just as accustomed to proper classroom etiquette and just as innocent of actually doing any mathematics. The major difference would be that our youthful scholars would be industriously engaged – indeed ofttimes furiously engaged – in the activity of note-taking.

On the face of it, note-taking is a rather curious sociological institution. One might imagine, from its name, that it consists in jotting down short phrases with which to jog the memory concerning important points of the professor's presentation. Memory, however, is hardly involved, for the whole point of the exercise is to copy down everything as accurately, as completely and as quickly as possible. Now, both the professor and the student have the textbook. The professor initiates the aforementioned sociological institution by copying the text onto sheaves of paper, which he/she then brings to class and from which he/she copies onto the blackboard. The student then takes up the cudgel by copying from the blackboard into his/her notebook. The circuit could be closed were the student to compare his/her notes to his/her copy (!) of the textbook, but, methinks, this is scarcely ever done.

Given the foregoing portrayal of what goes on in college mathematics classrooms, we might surmise that note-taking is a less than effective learning strategy. The present article, however, will have a narrower scope in that it will only demonstrate that note-taking is ineffective from the (radical) constructivist point of view. Before presenting the argument, however, it would behoove us to present some anecdotal evidence that is not limited to the constructivist perspective.

**One Student's Experience**

The course was Real Analysis. The first day of class was no different than any other: I assiduously took notes, just like everyone else. After class, though, I began to think things over. It was quite obvious, even from the first day, that our professor had a really nice way of explaining things. Too bad that it was not possible to appreciate his performance while straining to copy all the material that he was putting on the blackboard!

It then occurred to me, however, that he was presenting the material straight from the textbook. That bemused me for a while because I suddenly realized that it was a rather unprofitable enterprise to fill up my notebook, in my rather tremulous longhand, with the same theorems and exercises that were printed out so neatly in my textbook. Indeed, either the textbook or the notebook was superfluous – but which?

Thus, when the second day of class came merrily along, I resolved to forego the taking of notes, although I rather had a guilty conscious about doing so. Nevertheless, as the professor began his lecture, I resolutely sat at my desk, notebook open (lest I found the need to make a real note!), pen resting idly on the notebook's unbesmirched white page and I …, well, I was paying attention.

The result was all that could be hoped for. The lecture was as clear and distinct to me as a page of Aristotelian logic and I was able to interact with the professor about his material in an intelligent manner. At first, I must admit, I was a trifle reticent and would read along in the textbook as the professor wrote unconcernedly on the blackboard. Soon, however, I became bolder and closed the textbook so as to give my full attention to the professor's explanations. Indeed, it was not long before the unheard of happened: I began to read the text before coming to class. All and all, it became a wonderous experience.

I should not bring this anecdote to an end, however, without recounting how the professor and my fellow students reacted to my apparently insouciant behavior. It was never my intention to grandstand about this matter and so I went about it in a completely nondescript manner. But those who do not tow the line might just as well announce their novel behavior to the world with cymbals and the pounding of drums, because it is certain that he will be importuned. In the present case, all the same, harassment was not forthcoming from the professor. Although he indeed noticed from the beginning what I was doing, he was blithely, and perhaps wisely, unconcerned. Every once in a while, when he strayed from the straight and narrow, he would advise me that the theorem he was putting on the board was not included in the textbook and, thus, I might want to take notes on it. To these friendly counsels, I always concurred.

My classmates were possessed of much less discretion. In those first few days of that class, I was continuously, or so it seems, admonished by them of the recklessness of my actions and had to stoutly endure myriad predictions that I would fail the course. Suffice it to say that when the results of the first test came out, I suddenly found myself befriended by a number of benighted classmates who now saw me as an innovative learning resource.

**The Constructivist Classroom**

Our little anecdote does give us some *prima facie* evidence for looking askance at the activity of note-taking. It seems to indicate that, in immersing ourselves in the taking of notes, we disperse energy, attention and interest that could be put to more effective use elsewhere. Even so, anecdote is by definition anecdotal and, in the face of the long and perduring tradition of taking notes, one must wonder whether the student's experience, recounted above, was merely a serendipitous personal happenstance, or whether it is indicative of a generalizable phenomenon.

Perhaps it would be unwise to opine on the subject in the absence of empirical studies relating to the matter. Any empirical study, however, would be blind without the aid of theoretical insights with which it would have to be structured. Hence, we propose to investigate the question in relation to the constructivist theory of mathematics education. In particular, we will adopt herein the position of radical constructivism, as espoused by Ernst von Glasersfeld[1] and the researchers associated with him. Since this theory is well known, it will not be necessary to document each and every aspect of the theory we as we discuss it; rather, we may limit ourselves to culling from Fossa (2019) the principle aspects of the constructivist classroom and making some marginal references to the vast literature on constructivism.

In the aforementioned work, the constructivist classroom is contrasted with that of the traditional lecture approach to teaching. The major characteristics of the constructivist classroom, as gleaned from this work, are the following:

- student-centered classroom environment
- professor's role as authority figure is moderated
- dialogue and participant activities
- personal autonomy
- self-reliance
- real math being done by students
- richly interconnected cognitive space
- development of metacognitive processes
- traditional test format is replaced
- interpersonal relations are enhanced

[1] *Cf*., for example, von Glasersfeld (1991).

- positive social values are promoted.

We will presently entertain each of these characteristics in turn with a view towards investigating their implications regarding note-taking in the mathematics classroom.

**Student-centered Classroom Environment**

In the traditional lecture format, the professor is the star of the show. Everything is so arranged in order for the student to be able to see and hear clearly the professor's performance. In some lecture halls, this format is realized by having a raised dais for the professor to stand upon while he/she professes. This evidently makes eminent good sense, given the presuppositions of the traditional teaching method.

In point of fact, the basic idea informing the traditional lecture format is that the professor, by the means of his/her lecture, will transfer knowledge that he/she detains from him-/herself to the students. The medium of exchange is language. Thus, in order that the purported exchange be effected efficiently, it is imperative that the professor be in a relatively privileged position so that he/she can be observed without difficulty by all of his/her charges. In the traditional mathematics classroom, this translates into the student being so situated that he/she can see the blackboard and thereby be enabled to slavishly copy its contents into his/her notebook. We may also observe that there need be a certain decorum in the classroom since unruly behavior would undermine the conditions necessary for the transfer of knowledge.

In this setting, then, it is the professor that is the center of attention; indeed, it is he/she who usually talks and talks, while the student does not even listen because he/she is too busy scribbling. Much too often, as it turns out, the putative transfer of knowledge from the professor to the student is replaced by the sterile transfer of mathematical symbolism from the blackboard to the student's notebook. Note-taking, therefore, is detrimental to the success of the supposed transfer of knowledge even in the traditional lecture format.

Constructivism rejects the notion of the transfer of knowledge. It maintains that knowledge is constructed by each individual knower, albeit, I would argue, usually in a social context. The construction is effected by means of conceptual structures. This means that the student must be an active agent in the construction of his/her own knowledge structures; stated vulgarly, the student must think. He/she must think about the mathematics, try to make things fit together and build up his/her conceptual structures in a meaningful

way. If learning is to occur in the classroom, therefore, the classroom must be a student-centered learning environment, in which the student can be actively involved.

We have already seen, however, that the taking of notes tends to block effective thinking even in traditional classrooms. How much more, then, would it be anathema to the constructivist teacher, who strives to make the student's thought-constructions the centerpiece of his/her class. In short, note-taking is centering one's attention on the reproduction of someone else's thought, not on building up one's own conceptual structures. Note-taking, therefore, is not consonant with a student-centered classroom environment.

**The Professor as Authority Figure**

We have just seen that the professor holds an exalted position in the traditional classroom in that he/she is the center of everyone's attention. The same premises that endow the professor with such a status also makes him/her the classroom's authority figure – and this in two ways, *viz*., comportmentally and cognitively.

It is indeed seen to be the professor's responsibility to ensure that the proper conditions for learning obtain in his/her classroom. In this sense, the professor is invested with the authority to regulate the comportment of his/her charges.

The professor is also supposed to be a cognitive authority since it is he/she who detains the relevant knowledge which is to be imparted to the student.

Constructivism again challenges the concept of the authoritative professor, but in muted ways. Since the construction of cognitive structures obviates the necessity for the imposition of draconian regulation in order to insure the presumed transfer of knowledge, there is no need for the constructivist professor to have overwhelming comportmental authority. Nevertheless, there still remains an imperative for organizing classroom activities in a manner that will facilitate the achieving of pedagogical objectives.[2] To this end, the constructivist professor will act like a committee chairman, cajoling, not cudgeling the student.

Also, even a constructivist professor is a professor, not a student, because he/she should have a more richly structured and more valuable conceptual scheme than the student. It is his/her goal to facilitate the student's construction of schemes resembling his/her own. To do so, he/she acts like a guide, pointing out fruitful paths and warning of possible pitfalls.

[2] *Cf*. Davis, Maher and Noddings (1990).

Yet, it is the student who has to accept the guidance and use it to wind through the path for him-/herself.

How does note-taking relate to these concepts of authority? In the first place, it is easy to see that excessive note-taking is a marvelous disciplinary strategy. The student is obliged to spend all his/her energy in the busywork task of copying whatever the professor chooses to put on the blackboard, at whatever speed the professor chooses. This keeps the student occupied, quiet and "on task", albeit a mindless one. The strategy is all the more effective in that it does not present itself as a regulatory mechanism, but merely as classroom routine.

In the second place, taking notes from the supreme classroom cognitive authority renders the student impervious to the meaning of the mathematics. He/she is thereby encouraged to just "get it right" by straight memorization and is dissuaded from trying to figure out how mathematics is really built up, how different concepts fit together and, heaven forbid, to consider any alternatives.

In consequence, note-taking promotes an artificial posture of the professor as an unreasonable comportmental and cognitive classroom authority and is, therefore, not consonant with constructivism.

**Dialogue and Participant Activities**

Upon turning to "dialogue" and "participant activities", especially in the context of constructivism, one almost automatically thinks of ludic activities or, more generally, structured activities of the kind employed by mathematics educators in the primary grades. Games may be fine for grammar school, it may be argued, but at the college level it is more appropriate to use the lecture format.

Indeed, so goes the argument, the lecture reaches a large number of students efficiently. Moreover, college students may be considered adults and, thus, they would not be receptive to infantile methods of teaching and are intellectually prepared to assimilate the material presented in lectures. In response, we simply need observe the perceived necessity to take notes, often at a feverish pace, during which activity precious little assimilating is being done.

In any case, it would be ludicrous to identify "participant activities" with childish pursuits, for there are several college-level classroom teaching methods that can be classified as "participant activities" or that at least have high levels of student participation. Socratic

type questioning, the Moore Method and seminars, for example, all are of the requisite kind. These types of participant activities provoke student/teacher interactions and, perhaps to a lesser extent, student/student interactions about the mathematics being studied. The key to the success of these methods is maintaining a fruitful dialogue among the participants in the course.

In the lecture *cum* note-taking approach to mathematics teaching, there are certainly sundry student/professor interactions. Some examples are:

- "Professor, what's that word after 'continuous'?"
- "Professor, when's the first test going to be?"

The second of these queries is usually a spontaneous outburst, proffered in the middle of the professor's rehearsing of a complicated proof. They are obviously not part of a spirited dialogue about the mathematics being studied. In fact, the taking of notes inhibits dialogue because, as we have already seen, it inhibits thinking. Consequently, it is, with regard to the *desideratum* of fomenting classroom dialogues, inconsistent with constructivist principles.

**Personal Autonomy**

Much as what happened in the category of professorial authority, here too we must consider two cases, that of the autonomy of general life skills and that of the domain specific autonomy of the mathematics learner. The former is not always associated with the objectives of college-level instruction, since it may well be supposed that the general life skills of the student have already been infused into our hapless schoolchildren long before they ever see the inside of a college classroom. Be that as it may, or be that as it may not, a university education should, at the very least, deepen and develop those general skills that will stand the student in good stead in rest of his/her adult life.

For the constructivist, one (perhaps even the most) important educational goal is the development of personal autonomy.[3] This is, of course, a direct consequence of the constructivist's principle of knowledge construction by the individual, for, if the student is to be responsible for the construction of his/her own conceptual schemes, he/she must be free to make these constructions under his/her own providence. Construction is always a

[3] *Cf*. Confrey (1991).

series of choices and the constructivist proposes to assist the student in developing and/or strengthening the personal autonomy necessary for making those choices.

Note-taking, however, is antithetical to the development of personal autonomy. While taking notes, the student is wholly subservient to the professor and is rendered incapable of making any decisions – in sooth, all the choices are made by the professor and it is the professor that dictates even the pace of the student´s scribbling. If it reinforces any personal values, they seem to be none but those two values esteemed, by their superiors, in factory workers and soldiers, *viz*., obedience and punctuality.

With regard to mathematics education, the domain specific aspect of personal autonomy is the ability to undertake mathematical investigations with success, be those investigations one´s own original research or the critical appraisal of the work of others. Once again, it is a case of being able to make choices and work out for oneself the consequences of those choices.

It will behoove us to consider here the fact that mathematics is a social enterprise. This is important because there may seem to be a certain tension between autonomy and mutual cooperation. Scholastic philosophers undoubtedly would find a lovely niche here for their countless subtle distinctions and elusive definitions and, in fact, some of them would be helpful. When all has been parsed and logic-chopped into new and innovative categories, however, we still will find that sometimes certain tensions remain. What is nonetheless undeniable is that, in order to deal with those tensions in a creative and profitable manner, we must be truly autonomous mathematical agents.

Can we sustain, in a rational manner, that the taking of notes is a useful activity for the development of truly autonomous mathematical agents? Evidently not! While taking notes, the student is not capable of making any mathematical choices or of deciding how to proceed in a given proof, or even of appreciating the beauty of the mathematical structure. He/she is also incapable of catching a glimpse of the profound interrelations of ostensibly disparate parts of this ever-surprising intellectual enterprise. It is only after discontinuing the note-taking activity that he/she can do mathematically interesting things. But the professor, of course, is always ready to move on and any interlude in his/her presentation is just an opportunity to take a deep breath, preparatory to a new onrush. It is almost never an invitation to ponder.

Thus, once again, note-taking reveals itself in misalignment with the basic constructivist *Weltanschauung*. It inhibits the development of personal autonomy, both at the level of general life skills and at the level of mathematical competency.

This last, we must admit, comes as a surprise – and as a poignant indictment of pedagogical institutions that are more concerned with regulating the student that with educating him.

**Self-Reliance**

In many ways self-reliance is much akin to personal autonomy and, thus, we may expect a certain amount of overlap between the two categories, such as the existence of self-reliance both as a general life skill and as a domain specific value in mathematics education. Even so, the category of self-reliance is sufficiently nuanced to require consideration apart from autonomy.

Perhaps the most striking aspect of the category of self-reliance is the interaction between its general life skill and domain specific features. A person who is, generally speaking, self-possessed will have better chances of developing self-reliant behavior in the mathematics classroom and one's performance in mathematics may affect his/her general well-being. The student's in-class and out of class experiences, therefore, may set up a circular pattern of influences that may have either positive or negative reciprocal impacts.

One of the problems that mathematics educators must confront is that it is often considered to be an outrageously difficult subject and, thus, the student comes to it with great reserve. By the time the student reaches college-level mathematics such attitudes have become solidified and it is often extremely difficult to coax the student into thinking for him-/herself about mathematical concepts.

The problem outlined in the previous paragraph has a longitudinal complexity, which means that its solution stretches into a large portion of the student's lived experience and may require insights from various disciplines. We obviously cannot fully investigate the problem here. Nevertheless, we are obliged to consider how the seemingly innocent activity of note-taking affects the problem.

The first thing to observe is that note-taking is not a part of the solution. When the taking of notes is the student's principle activity in the mathematics classroom, he/she is afforded no opportunity for developing self-reliant attitudes. Self-reliance, indeed, involves not only the confidence that one can contribute to the success of a mathematical

investigation, but also that he/she can be wrong in intelligent, and therefore non-disparaging, ways. As long as the student can avoid having experiences of success and failure in mathematical thinking by squirreling him-/herself away behind his/her self-involved note-taking activities – or should we say, perchance, pseudo-activities –, he/she has no opportunity for developing self-reliant attitudes.

Before going on, we add the caveat that we have not herein addressed the important question of *how* to afford to the student positive experiences contributory to the development of his/her self-esteem. This is, as already indicated, a complex question, which, however, is not in the purview of the present work.

All the same, note-taking is not just omissive with regard to the development of self-reliance. It is also downright harmful. As long as the student is subjected to taking notes faithfully about things that he/she cannot completely understand, but which are subliminally consecrated by the double-barreled authority of the professor and the textbook, he/she must question his/her own abilities and is unable to see that his/her lack of understanding is not due to his/her own inadequacies, or to the inherent difficulties of the mathematics, but to the obscure fact that he/she has not been afforded the opportunity to think about the mathematical concepts that his/her notes purport to communicate to him/her. Worse, the aforementioned fact is "obscured", because it is hidden by the sleight of hand that, as we have already stated, presents note-taking as a normal – in fact, as the preferred – classroom activity.

Since the development of self-reliant, confidant and self-possessed attitudes, both with regard to the student's general life skills as well as his/her own mathematical abilities, is one of the principle goals of constructivist teaching theory, we must conclude that the taking of notes is not consonant with constructivism on this point.

**Real Math**

In order to understand what is meant herein by "real math", we must not oppose it to "unreal math", or, even worse, to "pure math". That is, we do not mean that real math is a type of mathematics that is somehow applied to the world and thereby obtains its significance. Rather, "real math" is to be opposed to "fake math" or to "make believe math", that is, to something that pretends to be mathematics, but that is not really mathematics.

Real math is not listening to somebody talk about mathematics or merely writing down mathematical statements or even mathematical demonstrations. It is not memorizing,

nor reciting, nor even "lecturing" about mathematics. Real math is doing mathematics, thinking about mathematical concepts, making them fit together in meaningful relationships and linking these mathematical structures to other, non-mathematical concepts.[4] It is solving mathematical problems, investigating mathematical hypotheses, considering possible counterexamples, trying to generalize and doing the various and sundry other things that real mathematicians do, if not on the frontiers of mathematical knowledge, at least on the frontiers of the student's mathematical knowledge.

For the constructivist, it is this real, dynamic mathematics, be it applied, or not, to physical interpretations, that should occupy the student during his classroom hours. Whether the student be a future professional mathematician, a worker for whom mathematics will be an important tool, or someone who just wants to appreciate the mathematical wonders that are part of the magnificent journey of human invention, there is absolutely no reason to deprive him/her of the experience of actually doing mathematics. In fact, the constructivist would point out that the only way of learning mathematics is by doing it.

Given all that has already been said in the present work, it should be evident that note-taking does not fit in well with real mathematics. Nevertheless, it bears repeating that, as long as the student is immersed in the mindless task of copying the professor's "lecture" from the blackboard, he/she is not doing any mathematics. The task of copying, which must be done quickly and with the utmost accuracy, is too intense to allow for any thinking to take place. At best, the notes may occasion mathematical thought if they are mulled over at one's leisure, but even this observation underscores the point that, as long as the student is taking notes, he/she is not doing any mathematics in the so-called mathematics classroom.

We may also observe that insofar as the professor's "lecture" is given over to copying material from the textbook onto the blackboard, he/she also is not doing any real mathematics. At first blush, this may seem to be a minor peccadillo on the professor's part, but in fact it is more serious than it at first appears, for one of the professor's roles as a teacher of mathematics is to model for the student real mathematical thinking; but this can only happen if the teacher fully engages with the mathematics.

[4] The term "real math" is used in this sense by Skemp (1989).

**Interconnected Cognitive Space**

Especially in relation to primary school mathematics, constructivists propose the establishment of classrooms with richly interconnected cognitive spaces which may promote reflective abstraction.[5] In the indicated context, this translates into the presence of an abundant amount of manipulative materials and structured activities.

Once we move on to the college-level mathematics classroom, as we have already mentioned, these kinds of manipulative materials become, for the most part, inappropriate. This does not mean, however, that richly interconnected cognitive spaces should be absent from the college classroom, but only that they take on different forms. Mathematics on the college level continues to abstract, but the "objects" that are abstracted from are no longer concrete objects from the physical universe; rather, they are themselves abstract objects, often constructed by prior mathematical activity.

It is not enough for the professor to pontificate on an abstract structure which has various interesting instances. These instances should be first familiar to the student before any abstraction be attempted. Next, the similarities among the various instances should be explored and only then should the generalization be attempted.

Once again, we see that the activity of taking notes does not allow for the development of the posited *desideratum* of establishing richly interconnected cognitive spaces in the classroom. There is no room for the student to do the kinds of exploration necessary for achieving this constructivist goal. At best, one may hope that the student, preferably in group study sessions, does this outside of the classroom and brings his/her questions back to the professor in subsequent classes. Unfortunately, however, in most cases the questions will be seen as distractions from the professor's program and, thus, he/she is likely to give but scant attention to them in order to press on to cover the day's planned material.

**Metacognition**

The concept of metacognition relates to all types of thinking in which the thinker is aware of and/or consciously monitors his/her own thought. Although it is characteristic of much higher order thinking, it is especially important in problem solving, in specific exploratory contexts and in delineating differences between the psychological behavior of

[5] *Cf*. Cooper (1991).

novices and experts. It is also clearly related to the constructivist concept of reflective abstraction.

Metacognitive processes are of great importance for most learning theories, be they constructivist or not. Thus, it is important to arrange the student's classroom activities in such a manner that the development of his/her metacognitive abilities may be fostered. Insofar, however, as the student is preoccupied with taking notes, he/she will not be involved in any activities that enhance metacognition. Consequently, from the point of view of the development of metacognition, the taking of notes is incompatible not only with constructivism, but most other theories of mathematics education as well.

**Test Formats**

One of the more controversial items in the constructivist theory of mathematics education is that of the role to be played by testing. Some kind of assessment of the student's learning is present in virtually all formalized educational contexts, but the type of testing employed and its underlying presuppositions vary greatly from one situation to another. Each educational institution may have its own policy on how the student's learning is to be evaluated and this may afford a certain regularity within the institution. Nevertheless, the professor is usually given much latitude in his/her choice of testing method and almost complete discretion in grading.[6]

In the traditional lecture-based format, assessment usually takes the form of individual sit-down, closed-book tests in which the student is required to solve problems that evidence his/her mastery of the mathematical techniques presented in the professor's lectures. The student may also be asked to demonstrate some propositions. These are usually fairly simple ones, designed to show a minimal understanding of the concepts involved, or ones whose proofs demand only minor variations in proofs showcased in those lectures.

Once again, all of this makes perfect sense, given the lecture format, which is based on the dubious presupposition that what goes on in the classroom is a transfer of knowledge to the student through the medium of language. Assessment is then rightly seen as an attempt to verify whether the putative transfer has occurred and/or the extent to which it has occurred.

Constructivists, as we have repeatedly seen, challenge the presupposition of the transfer of knowledge through language.[7] Rather, knowledge acquisition is seen as a

[6] For an overview of theories on testing in the mathematics classroom at the college level, see Kulm (1990).

[7] The present writer believes that the role of language in constructivist education is ofttimes sorely

continual process of the construction of mental schemes, built up into complex conceptual networks. Since the professor has no direct access to the student's mental schemes, he will want to use some indirect method of determining how the student has built up his/her conceptual network. Traditional tests, however, are woefully inadequate for making this determination because, unless they be accompanied by subsequent interviews in which the student explains to the teacher his/her thought processes for each answer, they cannot reveal the student's mental processes with any clarity. In consequence, constructivists tend to favor assessment techniques that are of a continual feedback nature, akin to certain psychological techniques used in the clinical interview.

Again, as long as the student is immersed in intense note-taking activities, he/she is refractory to probing with regard to any conceptual construction that may be taking place; in point of fact, such purported constructions are extremely unlikely in the given circumstances due to the very intensity and mind-numbing nature of the note-taking. Indeed, the professor him-/herself is completely occupied with his/her own scribbling at the blackboard and, thus, has little inclination, and no opportunity, to concern him-/herself about the student's understanding. Consequently, we must conclude that, with regard to assessment, the social institution of the taking of notes is inconsistent with constructivism.

**Interpersonal Relations**

When we think of the development of interpersonal relations, we admittedly usually think of the primary and secondary school, where this is a major educational objective. It is commonly assumed that the job is finished there and that by the time the student reaches the college level mathematics classroom, he/she has matured sufficiently for such juvenile objectives to be forgotten or, at least significantly downplayed.

As it turns out, the original (radical) constructivism of von Glasersfeld would probably be content to acquiesce in the outlook described in the previous paragraph, although there would still have to be some way for the professor to determine whether, or not, the student has successfully constructed the mental schemes that he/she has proposed as the objective of his teaching and he/she would still have to provide counterexamples to any inconsistent thought of the student, should that occur. Presumably, this could not be

misconstrued. Nonetheless, a thorough investigation of this question cannot be attempted here, but will be addressed on another occasion.

accomplished through the traditional test format and, therefore, some form of professor/student interaction would be called for.

Should we broaden our outlook, however, to include social constructivism, which developed historically in response to criticisms of the radical constructivist epistemology, we would recognize that mathematics, as indeed any other knowledge-based enterprise, is a social, not merely an individual, construction. Consequently, it would be imperative to foster not only professor/student, but also student/student interactions in order to create an ambience favorable to the constructive process.

Yet, as long as the student is locked into the activity of taking notes, he/she is practically isolated both from his/her classmates and the professor. All his/her attention is geared toward the blackboard and his/her notebook and there can be but little interpersonal interaction in the classroom, other than the minimally significant questioning of the professor's handwriting. Thus, once again we see that the taking of notes is not consistent with the constructivist paradigm, especially that of social constructivism.

**Positive Social Values**

The promotion of positive social values, such as the ability to cooperate with others in group projects, trust in others and friendship, are again usually seen to be objectives of the primary and secondary schools rather than college-level education. Nevertheless, some of the values that we have already seen to be important in the college-level mathematics classroom – autonomy and self-reliance – are bound up with those positive social values that we have just mentioned. Thus, there is at least an indirect fostering of these values by constructivists.

Even so, more need be said. The success of constructivist teaching methods is predicated on the development of dialogue and participant activities, as explained above. Such teaching methods depend on cooperation and trust which allows the student to externalize his/her thought processes and submit him-/herself to constructive criticism, both from the professor and his/her peers. This is often a harrowing experience for the student, especially when he/she knows that he/she is being judged – indeed his/her very adequacy as a person may seem to be in jeopardy along with his/her performance. It is imperative, therefore, that a safe space be created in the classroom, in which the student can articulate his/her thought in an honest an open manner and receive constructive criticism in a manner that will allow him/her to adjust his/her mental schemes in appropriate manners.

Were we to entertain social constructivist epistemologies, our conclusions would be strongly reinforced, since social constructivists contend that the very construction of knowledge is established socially and, therefore, positive social values are constitutive of effective learning processes.

As we have seen over and over again, note-taking is an individualized activity that not only does not promote the social aspect of knowledge construction, but also inhibits it due to its tendency to isolate the student from meaningful interactions with the professor and his/her fellow classmates.

**Partial Conclusion**

We may now sum up our investigation on the practice of note-taking in relation to the constructivist theory of mathematics education by citing an interesting article on the art of listening to the student in the classroom. The authors make insightful parallels between how one listens to students and how one goes about "listening to" original source material. To be a good listener, they argue, one must concur with the following three points:

> – Students are sense makers in idiosyncratically sophisticated ways,
> – Learning is a long-winded multifarious process in which social interactions (especially those between teacher and students) should respect ideas and incorporate them to the process, and that
> – The teaching role rests heavily in setting up and nurturing dialogues which are central to the ongoing reshaping of knowledge. (ARCAVI e ISODA, 2007, p. 127.)

Thus, the construction of knowledge is not a straightforward process, but one that is messy, complex and interactive. The student must continually negotiate his/her incipient conceptual schemes with those of the professor and his/her fellow classmates in order to gradually build up those conceptual structures constitutive of the mathematics to be learned.

Given these parameters, it is clear that the taking of notes is a pernicious classroom activity, in that, as we have seen in detail in the foregoing, it short-circuits the student's sense-making, isolates the student from any meaningful interaction with the professor, as well as his fellow classmates, and silences the dialogues necessary for the continual adjustment of his conceptual schemes.

**The General Setting**

Let us recall for a moment the anecdote with which the present work began. It should be clear that the classroom therein depicted was not governed by constructivist principles. Rather, it was a quite typical college classroom. Moreover, non-constructivist theories of mathematics education share with constructivism various *desiderata*, such as the development of strong metacognitive abilities. These observations lead us to reflect a bit on the role of note-taking from more general, non-constructivist perspectives.

It may be instructive to compare the typical lecture *cum* note-taking approach to graduate-level orientation. The former is notoriously unsuccessful, by all standards, in the educational enterprise, while the latter, despite the greater level of difficulty in the material taught, is much more fruitful. Just from the point of view of instruction, graduate courses tend to be much more individualized and dialogue-based – indeed, in the case of graduate orientation, it is wholly so.

Is it possible to introduce more individualized and dialogue-based instruction on the graduate level of mathematics instruction, given the larger class size and more diverse clientele? That this is indeed possible was pointed out already near the beginning of the present work. Instructional strategies such as the Moore Method or the organization of the classroom along the lines of a seminar would accomplish the stated purpose.

I would go so far, moreover, as to suggest that the professor could maintain his/her traditional method of lecture presentation and, nevertheless, improve the student's understanding of the mathematics taught by using the simple expedient of banning the practice of note-taking in his/her classroom. Of course, by doing so, it is likely that the professor would generate new kinds of student behavior, including increased questioning and thoughtful parallel investigations. This, in turn, would lead the professor to change some of his/her own behavior, resulting in richer and more successful instruction.

As a final note, I would like to relate another anecdote, or, rather, a felicitous historical encounter. It would seem that the illustrious German mathematician (Johann) Carl Friedrich Gauss (1777-1855) concurred with the position taken in the present work, since he "refused to let his pupils take notes, insisting that they would learn more by paying attention instead" (MUIR, 1962, p. 210). Although Muir misguidedly takes this as evidence that Gauss was an inferior teacher, we can only conclude that, as was true of his mathematics, he was ahead of his time in mathematics education as well.

## Conclusion

As a sociological institution, the lecture *cum* note-taking paradigm is ubiquitous in college-level mathematics classrooms. Nevertheless, it turns out that, from the perspective of (radical) constructivism, the taking of notes is antithetical to genuine learning. This conclusion was reached by looking at the constructivist position on the following eleven important issues: the establishment of a student-centered classroom environment, the professor's role as an authority figure, the use of dialogue and participant activities, the development of personal autonomy, the encouragement of self-reliance, the doing of real mathematics, the establishment of richly interconnected cognitive spaces in the classroom, the development of metacognitive abilities, the role of assessment, the enhancement of interpersonal relations and the promotion of positive social values. In all of these categories, note-taking was found to be inconsistent, to a greater or lesser extent, with constructivist teachings.

Moreover, since the educational goals of constructivism are shared with most other theories of mathematics education, there is considerable overlap between the *desiderata* and values embodied in the methods of instruction of constructivists and those of other theorists. Thus, in many respects, the constructivist critique of note-taking can be generalized to more generalized settings.

We conclude, therefore, that the taking of notes should be proscribed from the college-level mathematics classroom. Genuine note-taking, of course, would not be affected, where by "genuine note-taking" we mean jotting down succinct phrases to remind oneself of important points, as opposed to the obsessive copying of all the material that the professor puts on the blackboard. We may also observe that in some exceptional cases note-taking may be appropriate. Such might be the case when the student does not have access to textbooks. Even in these cases, however, we must recognize that there would be little learning going on in these classrooms, since the learning activities would be replaced with publishing activities and, thus, it would be imperative to employ other pedagogical strategies to help remedy the unfortunate situation.

## References

ARCAVI, Abraham, & ISODA, Masami. Learning to listen: from historical sources to classroom practice. **Educational Studies in Mathematics**, v. 66, p. 111-129, 2007.

CONFREY, Jere. What constructivism implies for teaching. In DAVIS, R. B.; MAHER, C. A.; NODDINGS, N. (Eds.), **Constructivist views on the teaching and learning of mathematics**. (**JRME Monograph Number 4**), p. 107-122, 1991.

COOPER, R. J. The role of mathematical transformations and practice in mathematical development. In: Steffe, L. P. (Ed.). **Epistemological foundations of mathematical experience**, p. 102-123, 1991.

DAVIS, R. B., MAHER, C. A., & NODDINGS, N. Introduction: Constructivist views on the teaching and learning of mathematics. In DAVIS, R. B.; MAHER, C. A.; NODDINGS, N. (Eds.), **Constructivist views on the teaching and learning of mathematics**. (**JRME Monograph Number 4**), p. 1-3, 1991.

FOSSA, J. A. **Intuitionist theory of mathematics education**. E-book at < www.researchgate.net/publication/331438081_Intuitionist_Theory_of_Mathematics_Education>, 2019.

KULM, Gerald. (Ed.) **Assessing higher order thinking in mathematics**. Washington, DC: American Association for the Advancement of Science, 1990.

MUIR, Jane. **Of men and numbers**. New York: Dell, 1962.

SKEMP, R. **Structured activities for primary mathematics**. London: Routledge, 1989.

VON GLASERSFELD, Ernst. An exposition of constructivism: why some like it radical. In DAVIS, R. B.; MAHER, C. A.; NODDINGS, N. (Eds.), **Constructivist views on the teaching and learning of mathematics**. (**JRME Monograph Number 4**), p. 19-29.

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