The LieCal Project and Its Investigation of Problem-Solving Strategies as a Measure of Longitudinal Curricular Effects on Students' Learning

O Projeto Local e sua Investigação de Estratégias de Solução de Problemas como uma Medida de Efeitos Curriculares Longitudinais sobre a Aprendizagem dos Alunos

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ABSTRACT

This paper presents a summary of the important middle school findings from the LieCal Project (Longitudinal Investigation of the Effect of Curriculum on Algebra Learning) and examines in detail the longitudinal effects of a middle school reform mathematics curriculum on students' open-ended problem solving in high school. Using assessment data from our large, longitudinal LieCal project, we compared the open-ended problem-solving performance and strategy use of high school students who had used the Connected Mathematics Project (CMP) in middle school with that of students who had used more traditional mathematics curricula. When controlling for sixth-grade state mathematics test performance, high school students who had used CMP in middle school had significantly higher scores on a multipart open-ended problem. In addition, high school students who had used CMP appeared to have greater success algebraically abstracting the relationship in the task.

Key Words: LieCal Project, Problem Solving, Curriculum Effect, Student Learning

RESUMO

Este artigo apresenta um resumo de importantes resultados relativos aos Anos Finais do Ensino Fundamental, obtidos a partir do projeto LieCal (Longitudinal Investigation of the Effect of Curriculum on Algebra Learning19) e examina, detalhadamente, os efeitos longitudinais que essa reforma do currículo de Matemática, neste nível de ensino, provoca nos estudantes do Ensino Médio, ao trabalharem com problemas abertos. Utilizar dados de avaliação deste amplo projeto longitudinal - LieCal, comparamos o desempenho e as estratégias utilizadas pelos alunos do Ensino Médio que tinham estudado Matemática através do CMP (Connected Mathematics Project20) nos Anos Finais do Ensino Fundamental com as de estudantes que vivenciaram um ensino de Matemática por meio de um currículo mais tradicional. Os testes de performance aplicados indicam que, em relação a um grupo de controle de alunos de 6º. ano, alunos do Ensino Médio que já haviam vivenciado o CMP no Ensino

[19] Pode ser traduzido como: Investigação longitudinal acerca dos efeitos do Currículo na aprendizagem de Álgebra.
[20] Projeto Matemática Conectada

Fundamental, apresentaram rendimento significantemente superior ao resolverem problemas abertos. Além disto, os alunos do Ensino Médio que haviam estudado matemática pelo CMP pareciam ter maior facilidade com as abstrações algébricas exigidas pelas tarefas.

Palavras-chave: Projeto LieCal, Resolução de Problemas, Efeitos Curriculares, Aprendizagem.

Introduction

Curriculum is a key lever for influencing the quality of education (BALL; COHEN, 1994; SENK; THOMPSON, 2003; VITHAL; VOLMINK, 2005). Therefore, educational researchers, practitioners, and policy-makers around the globe have sought to understand how to improve the curriculum and how to analyze the impact of curriculum reforms. In the United States, the National Council of Teachers of Mathematics (NCTM) was an early leader in providing recommendations for reforming and improving K-12 school mathematics through its *Standards* documents (1989, 2000). Among many recommendations about the goals for mathematics education, these and related documents emphasized the importance of engaging students in problem solving in the mathematics classroom. To make curricula that aligned with the NCTM standards available to teachers, the U.S. National Science Foundation (NSF) provided support to develop a number of so-called *Standards*-based school mathematics curricula for elementary, middle, and high school students. With the implementation of these curricula came the need to assess their effectiveness at achieving the goals set out in the various standards documents, including their effectiveness at helping students become effective mathematical problem solvers (see Senk; Thompson, 2003, for an overview of the assessments of the NSF-funded curricula).

In this paper, we report on results from the *Longitudinal Investigation of the Effect of Curriculum on Algebra Learning* (LieCal). The LieCal Project sought to longitudinally investigate the effects of the Connected Mathematics Project (CMP), one of the NSF-funded *Standards*-based curricula. The CMP curriculum is a complete middle-school mathematics program, and can be characterized as a problem-based curriculum whose intent is to build students' understanding in the four mathematical strands of number and operations, geometry and measurement, data analysis and probability, and algebra through explorations of real-world situations and problems (LAPPAN et al., 2002b). The LieCal project compared the effects of the CMP curriculum with those of traditional, non-CMP middle school mathematics curricula, both within the middle school grades and into high school. The present study examines the open-ended problem-solving performance and strategy use in high school of former CMP and non-CMP middle school students.

Research on Mathematical Problem Solving

There is a long history of interest in integrating problem solving into school mathematics (CAI; NIE, 2007; SIU, 2004; STANIC; KILPATRICK, 1989). Research in mathematics education has correspondingly attended to multiple aspects of mathematical problem solving and the ways in which problem solving can play a role in school mathematics, ranging from how students learn mathematics, to how teachers teach and assess

students' learning of mathematics, to how mathematics is presented in curricula. Looking across the scholarship on mathematical problem solving, six interrelated lines of research stand out.

The first line of research seeks to shed light on the processes involved in problem solving (e.g., FRENSCH; FUNKE, 1995; LESH; ZAWOJEWSKI, 2007; LESTER, 2013; MCLEOD; ADAMS, 1989; SCHOENFELD, 1992; SILVER, 1985). Problem solving is inherently a complex process, and researchers working in this line of research have attended to diverse aspects of the problem solving process, notably affective, cognitive, and metacognitive aspects.

The second line of problem-solving research focuses on the teaching of mathematical problem solving in classrooms (CAI, 2003, 2010; KROLL; MILLER, 1993; LESH; ZAWOJEWSKI, 2007; WILSON et al., 1993). Researchers working in this area have investigated the teaching of mathematics with a focus on problem solving (e.g., HEMBREE; MARSH, 1993; HENNINGSON; STEIN, 1997; HIEBERT et al., 1997; KROLL; MILLER, 1993; STEIN; SMITH; SILVER, 1999). In this work, problem solving is viewed as a learning goal of school mathematics – the aim is to improve students' success at solving problems. This type of teaching is usually called problem-solving instruction, and it has been studied extensively. Cai (2010) and Lester and Cai (2016) have conducted reviews of research in this area.

The third line of problem-solving research is also concerned with teaching, but differs from the previous line of research in that problem solving itself is not the learning goal. Rather, problem solving is a means for teaching mathematics (LESTER; CHARLES, 2003; SCHOEN; CHARLES, 2003; SCHROEDER; LESTER, 1989). Compared to the research on problem-solving instruction, the line of research on teaching mathematics *through* problem solving focuses on a relatively new idea in the history of problem solving in the mathematics curriculum. Even though teaching mathematics through problem solving is a rather new conception, there is widespread agreement that teaching through problem solving holds great promise for fostering student learning (CAI, 2003).

A fourth line of research on problem solving is related to problem posing, which has been recognized as a component of the problem-solving process (CAI; HWANG, 2002; CAI et al., 2013; CAI et al., 2015; SINGER; ELLERTON; CAI, 2015; SILVER, 1994). Although problem-posing research is a relatively new endeavor, it has prospered in recent years. Indeed, there have been efforts to incorporate problem posing into school mathematics at different educational levels around the world.

A fifth line of mathematical problem-solving research is related to research on mathematical modeling, as the modeling process can be viewed as a specific kind of problem solving. Research on mathematical modeling has taken a number of perspectives, including mathematical, cognitive, curricular, instructional, and teacher education (e.g., CAI et al., 2014).

Finally, a sixth line of research on problem solving focuses on using both problem solving and problem posing for the assessment of students' learning (e.g., CAI et al., 2013). The research reported here is of this type. Studies of problem solving in mathematics education have already moved from a focus only on the product (i.e., the actual solution to the problem) to a focus on the process (i.e., the set of planning and executing activities that direct

the search for a solution). Individual differences in solving mathematical problems can sometimes be understood in terms of differences in students' uses of various strategies. Proficiency in solving mathematical problems is dependent on the acquisition, selection, and application of both domain-specific strategies and general cognitive strategies (SCHOENFELD, 1992; SIMON, 1979). Thus, competence in using appropriate problem-solving strategies reflects a high degree of performance and proficiency in mathematics. In fact, researchers have long used the examination of problem-solving strategies to assess and evaluate instructional programs and education systems (CAI, 1995; FENNEMA et al., 1998). Therefore, using problem solving to assess mathematics proficiency implies that effective problem-solving assessment tasks should be designed to reveal the various strategies that students employ. Moreover, students' problem-solving strategies can become more effective over time. Therefore, both the examination of the strategies that students apply and the success of those applications can provide information regarding the developmental status of students' mathematical thinking and reasoning.

Whether problem solving is viewed as a process, a learning goal, an instructional approach, modeling, or a means of assessment, it is clear from the research that problem solving should be an integral part of mathematics learning, and a significant commitment should be made to include problem solving at every grade level and with every mathematical topic. A review by Cai (2010) showed that teachers should engage students in a variety of problem-solving activities in order to help students become successful problem solvers and also learn mathematics better through: (1) finding multiple solution strategies for a given problem, (2) engaging in problem posing and mathematical exploration, (3) giving reasons for their solutions, and (4) making generalizations. Cai's review also showed that focusing on problem solving in the classroom not only impacts the development of students' higher-order thinking skills, but also reinforces positive attitudes.

Our findings, which are related to the sixth line of research about problem solving, come from the LieCal project's longitudinal examination of the effect of CMP and non-CMP curricula on students' mathematics learning. The purpose of the present study is to use problem solving as a measure to longitudinally examine the effect of a problem-based curriculum on students' learning of mathematics. More specifically, this study uses the examination of students' problem solving strategies to investigate how the use of different types of middle school curricula affects the learning of high school mathematics for a large sample of students from ten high schools in an urban school district.

The LieCal Project

CMP and Non-CMP Curricula

In examining and understanding the differential effects of the CMP and non-CMP curricula on students' problem-solving performance and strategies, it is necessary to consider the ways in which these curricula diverge in their treatment of key algebraic concepts. An examination of the CMP and non-CMP curriculum materials show clear differences in this regard. In particular, the curricula make use of strikingly different conceptions about algebra – a functional approach in the CMP curriculum and a structural approach in the non-CMP curricula. Below, we describe several examples that illustrate these different conceptions of

and approaches to algebra.

Defining and introducing the concept of variables.

Because of the central role of variables in algebra, the contrasting ways in which the CMP and non-CMP curricula introduce variable ideas are of particular note (NIE et al., 2009). The learning goals of the CMP curriculum characterize variables as the representations of quantities in relationships. Though the CMP curriculum does not formally define "variable" until 7th grade, CMP's informal characterization of a variable as a quantity that changes or varies makes it convenient to use variables informally to describe relationships long before the formal introduction of the concept of variables in 7th grade. The choice to define variables in terms of quantities and relationships reflects the functional approach that the CMP curriculum takes.

In contrast, the learning goals in the non-CMP curricula characterize variables as placeholders or unknowns. The non-CMP curricula formally define "variable" in 6th grade as a symbol (or letter) used to represent a number. Variables are treated predominantly as placeholders and are used to represent unknowns in expressions and equations. By introducing the concept of variables in this fashion, the non-CMP curricula support a structural approach to algebra.

Defining and introducing the concept of equations.

Given the functional approach to variables in the CMP curriculum and the structural approach in the non-CMP curricula, it is not surprising that the concept of equation is similarly defined functionally in CMP, but structurally in the non-CMP curricula. In CMP, equations are a natural extension of the development of the concept of variable as a changeable quantity used to represent relationships. At first, CMP expresses relationships between variables with graphs and tables of real-world quantities rather than with algebraic equations.

Later, when CMP introduces equations, the emphasis is on using them to describe real-world situations. Rather than seeing equations simply as objects to manipulate, students learn that equations often describe relationships between varying quantities (variables) that arise from meaningful, contextualized situations (BEDNARZ; KIERAN; LEE, 1996). In the non-CMP curricula, the definition of a variable as a symbol develops naturally into the use of context-free equations with the emphasis on procedures for solving equations. These are all hallmarks of a structural focus. For example, one non-CMP curriculum defines an equation as "…a sentence that contains an equals sign, =" illustrated by examples such as $2 + x = 9, 4 = k - 6$, and $5 - m = 4$. Students are then told that the way to solve an equation is to replace the variable with a value that results in a true sentence.

Defining and introducing equation solving.

In line with their treatment of variables and equations, the means by which the CMP and non-CMP curricula introduce equation solving reflect functional and structural approaches, respectively. In the CMP curriculum, equation solving is introduced within the context of discussing linear relationships between quantities. The initial treatment of equation solving does not involve symbolic manipulation, as found in most traditional curricula.

Instead, the CMP curriculum introduces students to linear equation solving by using a graph to make visual sense of what it means to find a solution. Its premise is that a linear equation in one variable is, in essence, a specific instance of a corresponding linear relationship in two variables. It relies heavily on the context in which the equation itself is situated and on the use of a graphing calculator.

After CMP introduces equation solving graphically, the symbolic method of solving linear equations is finally broached. It is introduced within a single contextualized example, where each of the steps in the equation-solving process is accompanied by a narrative that demonstrates the connection between what is happening in the procedure and in the real-life situation. In this way, CMP justifies the equation-solving manipulations through contextual sense-making of the symbolic method. That is, CMP uses real-life contexts to help students understand the meaning of each step of the symbolic method of equation solving, including why inverse operations are used. As with the introduction of variables and equations, CMP's functional approach to equation solving maintains a focus on contextualized relationships among quantities. Figure 1 below shows an example of equation solving in the CMP curriculum.

The Unlimited Store allows any customer who buys merchandise costing over $30 to pay on the installment plan. The customer pays $30 down and then pays $15 a month until the item is paid for. Suppose you buy a $195 CD-ROM drive from the Unlimited Store on an installment plan, How many months will it take you to pay for the drive? Describe how you found your answer.

Thinking	**Manipulating the Symbols**
"I want to buy a CD-ROM drive that costs $195. To pay for the drive on the installment plan, I must pay $30 down and $15 a month."	$195 = 30 + 15N$
"After I pay the $30 down payment, I can subtract this from the cost. To keep the sides of the equation equal, I must subtract 30 from both sides	$195 – 30 = 30 – 30 + 15N$
"I now owe $165, which I will pay in monthly installments of $15."	$165 = 15N$
"I need to separate $165 into payments of $15. This means I need to divide it by 15. To keep the sides of the equation equal, I must divide both sides by 15."	$\frac{165}{15} = \frac{15N}{15}$
"There are 11 groups of $15 in $165, so it will take 11 months."	$11 = N$

Figure 1.
An Example of Equation Solving in CMP (LAPPAN et al., 2002a, p. 55).

In the non-CMP curricula, contextual sense-making is not used to justify the equation-solving steps as it is in the CMP curriculum. Rather, the non-CMP curricula first introduce equation solving as the process of finding a number to make an equation a true statement. Specifically, *solving* an equation is described as replacing a variable with a value (called the *solution*) that makes the sentence true. Equation solving is introduced in the non-CMP curricula symbolically by using the additive property of equality (equality is maintained if the same quantity is added to or subtracted from both sides of an equation) and the multiplicative property of equality (equality is maintained if the same non-zero quantity is multiplied by or divided into both sides of an equation). This approach to equation solving is aligned with the non-CMP curricula's structural focus on working abstractly with symbols and procedures.

Cognitive demand of mathematical problems.

In the LieCal Project, the cognitive demand of mathematical problems in both the CMP curriculum and a representative non-CMP curriculum were analyzed (Cai, Nie, & Moyer, 2010). The problems were classified into four increasingly demanding categories of cognition: memorization, procedures without connections, procedures with connections, and doing mathematics (Stein & Lane, 1996). As Figure 2 illustrates, the CMP curriculum had significantly more high-level tasks (procedures with connections or doing mathematics) ($\chi^2(3, N = 3311) = 759.52, p < .0001$) than the non-CMP curricula. This kind of analysis of the intended curriculum provides insight into the degree to which different curricula expect students to engage in higher-level thinking and problem solving.

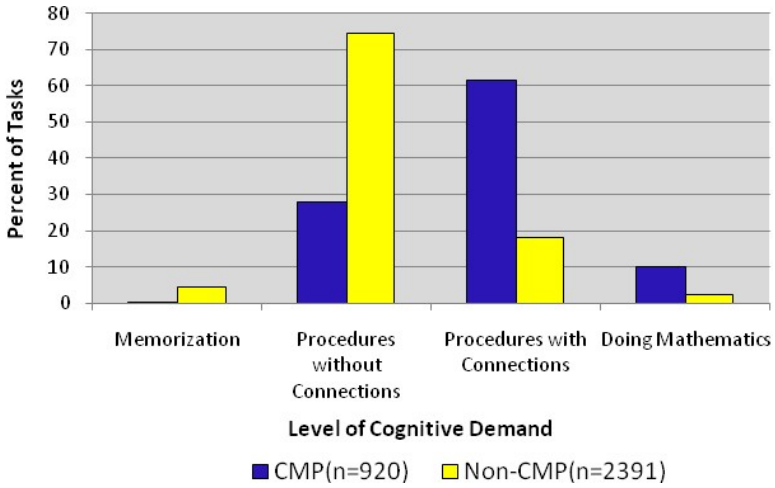

Figure 2.The percentage distributions of the cognitive demand of the instructional tasks intended in the CMP and non-CMP curricula.

Instruction in CMP and Non-CMP Classrooms

In order to better understand how the differences in the CMP and non-CMP curriculum materials play out in actual classrooms, the LieCal Project collected data on multiple aspects of implementation based on 620 detailed lesson observations of CMP and non-CMP lessons over a three-year period. Approximately half of the observations were of teachers using the CMP curriculum, while the other half were observations of teachers using non-CMP curricula. Two retired mathematics teachers conducted and coded all the observations. The observers received extensive training that included frequent checks for reliability and validity throughout the three years (MOYER et al., 2011; NIE et al., 2013).

Each class of LieCal students was observed four times, during two consecutive lessons in the fall and two in the spring. The observers recorded extensive information about each lesson using a 28-page project-developed observation instrument. During each observation, the observer made a minute-by-minute record of the lesson on a specially designed form. This record was used later to code the lesson. The coding system had three main components: (1) the structure of the lesson and use of materials, (2) the nature of the instruction, and (3) the analysis of the mathematical tasks used in the lesson.

The analyses of the data obtained from the classroom observations revealed striking differences between classroom instruction using the CMP and non-CMP curricula. Below, we

briefly review the differences that were related to three important instructional variables that could have an impact on students' problem-solving: (1) the level of conceptual and procedural emphases in the lessons, (2) the cognitive level of the instructional tasks implemented, and (3) the cognitive level of the assigned homework problems.

Conceptual and procedural emphases.

The second component of the coding section included twenty-one 5-point Likert scale questions that the observers used to rate the nature of instruction in a lesson. Of the 21 questions, four were designed to assess the extent to which a teacher's lesson had a conceptual emphasis. Another four questions were designed to determine the extent to which the lesson had a procedural emphasis. Factor analysis of the LieCal observation data confirmed that the four procedural-emphasis questions loaded on a single factor, as did the four conceptual-emphasis questions.

There was a significant difference across grade levels between the levels of conceptual emphasis in CMP and non-CMP instruction ($F = 53.43$, $p < 0.001$). The overall (grades 6-8) mean of the summated ratings of conceptual emphasis in CMP classrooms was 13.41, whereas the overall mean of the summated ratings of conceptual emphasis in non-CMP classrooms was 10.06. Since the summated ratings of conceptual emphasis were obtained by adding the ratings on the four items of the conceptual-emphasis factor in the classroom observation instrument, the mean rating on the conceptual-emphasis items was 3.35 (13.41/4) for CMP instruction and 2.52 (10.06/4) for non-CMP instruction. That is, CMP instruction was rated 0.40 points above the midpoint, whereas non-CMP instruction was rated 0.5 points below the midpoint. Thus, on average, CMP instruction was rated about 4/5 of a point higher (out of 5) on each conceptual emphasis item than non-CMP instruction, which was a significant difference ($t = 11.44$, $p < 0.001$).

In contrast, non-CMP lessons had significantly more emphasis on the procedural aspects of learning than the CMP lessons. The procedural-emphasis ratings for the non-CMP lessons were significantly higher than the procedural-emphasis ratings for the CMP lessons ($F = 37.77$, $p < 0.001$). Also, the overall (grades 6-8) mean of summated ratings of procedural emphasis in non-CMP classrooms (14.49) was significantly greater than the overall mean of the summated ratings of procedural emphasis in CMP classrooms, which was 11.61 ($t = -9.43$, $p < 0.001$). Since the summated ratings of procedural emphasis were obtained by adding the ratings on the four items of the procedural-emphasis factor, the mean rating on the procedural emphasis items was 3.62 (14.49/4) for non-CMP instruction and 2.91 (11.61/4) for non-CMP instruction. On average, non-CMP instruction was rated about 7/10 of a point higher (out of 5) on each procedural emphasis item than CMP instruction, which was a significant difference.

Instructional tasks.

As was done (above) with the mathematical problems in the intended curricula, the scheme developed by Stein et al. (1996) was again used to classify the instructional tasks actually used in the CMP and non-CMP classrooms into four increasingly demanding categories of cognition: memorization, procedures without connections, procedures with connections, and doing mathematics. Figure 3 shows the percentage distributions of the

cognitive demand of the instructional tasks implemented in CMP and non-CMP classrooms (note that Figure 2 referred to problems from the intended, not the implemented, curricula).

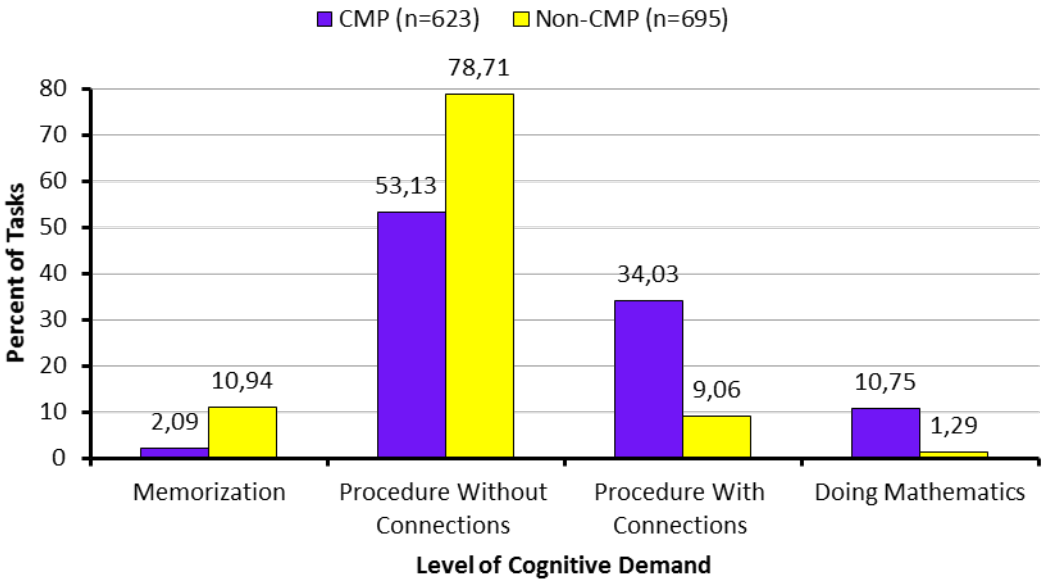

Figure 3.
The percentage distributions of the cognitive demand of the instructional tasks implemented in the CMP and non-CMP classrooms

The percentage distributions in CMP and non-CMP classrooms were significantly different ($X^2(3, N = 1318) = 219.45$, $p < .0001$). The difference confirms that a larger percentage of high cognitive demand tasks (procedures with connection or doing mathematics) were implemented in CMP classrooms than were implemented in non-CMP classrooms ($z = 14.12$, $p < .001$). Moreover, a larger percentage of low cognitive demand tasks (procedures without connection or memorization) were implemented in non-CMP classrooms than were implemented in CMP classrooms. In addition, not only did CMP teachers implement a significantly higher percentage of cognitively demanding tasks than non-CMP teachers across the three grades, but also within each grade (z values range from 6.06 – 11.28 across the three grade levels, $p < .001$).

Student Achievement for CMP and Non-CMP Students

In the LieCal Project, we have examined how the use of the CMP and non-CMP curricula have produced differing profiles of student mathematics performance. Looking within the middle school grade band, the LieCal Project found that on open-ended tasks assessing conceptual understanding and problem solving, the growth rate for CMP students over the three middle school years was significantly greater than that for non-CMP students (Cai et al., 2011). At the same time, CMP and non-CMP students showed similar growth over the three middle school years on the multiple-choice tasks assessing computation and equation-solving skills. These findings suggest that the use of the CMP curriculum is associated with a significantly greater gain in conceptual understanding and problem solving than is associated with the use of the non-CMP curricula. However, those relatively greater conceptual gains do not come at the cost of lower basic skills, as evidenced by the comparable

results attained by CMP and non-CMP students on the computation and equation solving tasks.

The LieCal Project subsequently followed the students into their high school years. All high schools in the district are required to use the same district-adopted mathematics curriculum. CMP and non-CMP students were mixed into each class in each of ten high schools in the same district. Thus, all of the former CMP and non-CMP students used the same curriculum in high school and were taught by the same teachers in their high schools. As an extension of the results we found in the middle school years, we have been examining whether the superior problem-solving abilities gained by the CMP students in middle school result in better performance on a delayed assessment of mathematical problem solving in high school.

In a previous study, we used problem posing as a measure of middle school curricular effect on students' learning in high school (CAI et al., 2013). Using problem posing as a measure, we found that in high school, students who had used the CMP curriculum in middle school performed equally well or better than students who had used more traditional curricula. The findings from this previous study not only showed evidence of the strengths one might expect of students who used the CMP curriculum, but also demonstrated the usefulness of employing a qualitative rubric to assess different characteristics of students' responses to the posing tasks. Moreover, given the potential role of problem posing within the problem-solving process, this result suggests that the former CMP students might also continue to exhibit enhanced problem-solving strategies and performance in high school than their non-CMP counterparts. Thus, in the present study we use open-ended problem-solving strategies as a measure to examine longitudinal curricular effect on students' learning.

Problem-Solving Strategies as a Measure of Longitudinal Curricular Effects on Students' Learning

Methods

Participants

In the LieCal Project, we followed more than 1,300 students (650 using CMP and 650 using non-CMP curricula) from a school district in the United States for three years as they progressed through grades 6-8. In the 2008-2009 school year, most of these 1,300 CMP and non-CMP students from the middle school study entered high schools as freshmen. We then followed the students enrolled in the 10 high schools that had the largest numbers of the original 1,300 CMP and non-CMP students. As noted above, the former CMP and non-CMP students were mixed into high school classes that used the same curriculum.

Assessment Tasks and Analyses

As part of the LieCal Project, we developed and used 13 open-ended tasks to assess students' learning in high school, specifically the 11th and 12th grades. Students' responses were analyzed in two ways. The first was to quantitatively score each student response using

a previously-developed holistic scoring rubric. The second was to qualitatively analyze students' responses with a focus on their solution strategies. In this paper, we mainly draw on results from an analysis of solution strategies to a pattern problem called the doorbell problem (see Appendix). This five-part task assesses students' ability to find regularities of a pattern and make generalizations. We chose to report the results from this task because it is representative of the tasks we used to assess students' generalization skills.

Data Collection and Coding

As part of the larger longitudinal study, we assessed 533 students (321 CMP and 212 non-CMP) in the fall of 11th grade (Fall, 2010), spring of 11th grade (Spring, 2011), and spring of 12th grade (Spring, 2012). The data for the analyses of students' strategies came mainly from the 12th grade spring assessment. In a small number of cases, if a student did not participate in the Spring 2012 assessment but did participate in the Spring 2011 assessment, we used the data from the Spring 2011 assessment. If a student did not participate in either the Spring 2012 or Spring 2011 assessments, but had participated in the Fall 2010 assessment, we used the data from the Fall 2010 assessment. This allowed us to look at the students' most recent attempt at each task.

As noted above, students' responses to the doorbell problem were first scored using a holistic scoring rubric that took into account the students' numerical answers and their explanations of their strategies. The responses were then also qualitatively coded for the types of strategies used. We coded students' solution strategies for parts A, B, C, and E as an abstract strategy, a concrete strategy, an unidentifiable strategy, or no strategy. Students who used an abstract strategy were able to recognize that the number of guests entering for each ring was equal to either two times the ring number minus one (i.e., $y = 2n - 1$) or the ring number plus the ring number minus one (i.e., $y = n + (n - 1)$). Students who used a concrete strategy were able to identify that the number of guests who enter increases by two for each doorbell ring and then sequentially adding two until they reached the desired number of rings, but did not abstract an algebraic formula. An unidentified solution strategy was a strategy that did not particularly make sense for the problem (e.g., $y = [r(100) + 2] - 1$). Lastly, a student was said to have used no strategy if the student did not show work for his or her answer, or if he or she did not attempt to answer the question at all.

Students' strategies for part D were coded in one of five ways. First, the student could have completely abstracted the algebraic formulas $2n - 1$ or $n + (n - 1)$. Secondly, they could have completely abstracted the pattern in a verbal description (e.g. "The number of guests who entered on a particular ring of the doorbell equalled two times that ring number minus one."). Third was an incomplete abstraction that only captured a recursive relationship, such as, "When the bell rings, two more people come." Fourth was an unidentified strategy, which either represented the strategies for students who incorrectly answered the question or had a provided a strategy that did not make sense. Finally, a strategy was coded as "no strategy" if no attempt was made to solve the problem.

Results

Overall Performance on the Doorbell Problem

We first conducted analyses based on the quantitative scoring (using holistic scoring rubric that took into account the students' numerical answers and their explanations of solution strategies) to student responses to the doorbell problem. The analyses indicated significant curriculum effects under two covariates for the doorbell problem. When controlling for overall state math test exam scores for 6th grade, CMP students scored significantly higher than non-CMP students on the doorbell problem ($t = 2.09$, $p = 0.0371$). When controlling for scores on the algebra subtest on the overall state math test for 6th grade, CMP students still scored significantly higher than non-CMP students ($t = 2.47$, $p = 0.0141$).

Performance on Individual Parts of the Doorbell Problem

Chi-squared tests were performed to look for relationships between curriculum and correctness of answers on each part of the doorbell problem. For part A, there was a significant relationship between curriculum and correct answers ($\chi^2 = 6.5363$, $p < 0.040$). That is, a significantly larger percentage of the CMP students had correct answers than the non-CMP students. For parts B, C, D, and E, there were no significant relationships between curriculum and correct answers. For each of the five parts of the problem, Table 1 provides the percentage of students with correct answers in that part. Note that Table 1 shows a considerable decreasing trend in the number of students who found a correct solution from part A to part E.

	Doorbell Problem Part				
Curriculum	A	B	C	D	E
CMP ($n = 321$)	80.1	38.6	27.7	18.1	7.5
Non-CMP ($n = 212$)	74.5	35.4	27.4	16.0	5.2

Table 1
Percentages of CMP and non-CMP students who correctly solved each part of the Doorbell Problem

Concrete and Abstract Solution Strategies

Focusing specifically on the solution strategies of those students who provided correct solutions for parts of the doorbell problem, the results were mixed. For part A (see Table 2), 67.3% of CMP students ($n = 257$) and 63.9% of non-CMP students ($n = 158$) used a concrete strategy to find the correct answer, whereas 26.1% of CMP students and 27.8% of non-CMP students abstracted the problem to an algebraic formula. There were no significant differences in proportion between CMP and non-CMP students for each strategy.

However, some differences in strategy use arose between the two groups as well. For part B (see Table 2), 73.4% of CMP students (n=124) and 60% of non-CMP students (n=75) abstracted the problem to an algebraic formula to find the correct solution, whereas 17.7% of CMP students and 24.0% of non-CMP students used a concrete strategy. A significantly greater proportion of CMP students used the abstract strategy than did the non-CMP students ($z = 1.97$, $p < 0.050$), but there was no significant difference in proportion between CMP and non-CMP students for the concrete strategy.

For part C (see Table 2), 71.9% of CMP students (n=89) and 67.2% of non-CMP students (n=58) abstracted the problem to an algebraic formula, whereas 7.9% of CMP students and 19.0% of non-CMP students used concrete strategies to find a correct solution. A significantly greater proportion of non-CMP students used the concrete strategy than did the CMP students (z = -2.27, p < 0.025), but there was no significant difference in proportion between CMP and non-CMP students for the abstract strategy.

		Type of strategy (%)			
Problem part	*N*	Abstract	Concrete	Unidentified	None
A					
CMP	257	26.1	67.3	3.5	3.1
Non-CMP	158	27.8	63.9	1.9	6.3
B					
CMP	124	73.4	17.7	3.2	5.6
Non-CMP	75	60.0	24.0	4.0	12.0
C					
CMP	58	71.9	7.9	9.0	11.2
Non-CMP	34	67.2	19.0	5.2	8.6
D					
CMP	58	100.0	0.0	0.0	0.0
Non-CMP	34	100.0	0.0	0.0	0.0
E					
CMP	24	62.5	29.2	4.2	4.2
Non-CMP	11	45.5	36.4	0.0	18.2

Table 2:
Percentages of CMP and non-CMP students who used each type of strategy to correctly answer parts of the doorbell problem

For part D, almost every student who provided a correct solution responded in nearly the same way. All of the 34 non-CMP students and 54 out of 58 CMP students who correctly answered this part generated an algebraic abstraction and provided a mathematical formula. The remaining four CMP students wrote out a verbal description of the mathematical formula, which would still require them to have first abstracted the relationships before translating those relationships into written form.

Part E seemed to be a challenging question for both the CMP and non-CMP students. Only 24 CMP students and 11 non-CMP students provided a correct solution to this part of the doorbell problem. Given these small sample sizes, although there were noticeable group differences in raw percentages of students using algebraic and concrete strategies, with a greater proportion of CMP students than of non-CMP students using algebraic strategies, these differences were not statistically significant.

Discussion

As part of a larger longitudinal study of curricular effect on mathematics learning, the results we have presented above provide a useful perspective on the potential long-term impacts of reform mathematics curricula on students' mathematical thinking and problem solving. Although we have presented data from only one open-ended task, the results suggest that high school students who used the CMP curriculum in middle school were more

successful than their peers who used more traditional middle-school curricula at solving the doorbell problem and explaining their solution strategies. This result aligns with those obtained when these students were still in middle school (CAI, et al., 2011). The result is also consistent with our previous findings using problem posing as measure of curricular effect (CAI et al., 2013). Thus, it would appear that the CMP students' problem-solving gains persist well into high school.

The retention of these gains over longer time intervals also parallels the findings from research on the effectiveness of problem-based learning (PBL) in medical education (HMELO-SILVER, 2004). In that context, medical students trained using a PBL approach performed better than non-PBL students on conceptual understanding and problem-solving ability even when assessed at a later time. In a similar fashion, the CMP students in the LieCal project experienced problem-based instruction that focused on developing students' conceptual understanding and problem solving abilities.

In addition, our analysis of the strategies used by the students in this study suggests that the CMP students who correctly solved the parts of an open-ended task were somewhat more likely to make generalizations. This appears to reflect the emphasis in the CMP curriculum on relationships between quantities (i.e., the functional approach). The ability to abstract algebraic relationships from real-world situations appears to also have persisted in the CMP students.

Note that for this analysis, we focused on the strategies of students who correctly answered one or more parts of the doorbell problem. We did not consider the strategies of students who failed to provide correct answers. Future work will include additional analyses to probe the strategies of students who provided incorrect answers to the doorbell problem parts, as well as analyses of student response to other open-ended problems.

Conclusion

Mathematical problem solving continues to be a key feature of mathematics curricula, and consequently a focus of mathematics education research. As we noted at the beginning of this paper, research on mathematical problem solving has pursued many different aspects, including the cognitive processes of problem solving, the teaching of problem solving, components of the problem-solving process including problem posing, mathematical modeling, and the use of problem solving and posing as assessments of students' mathematical learning. The study we have reported in this paper stems from this final line of research into problem solving as assessment. We have explored how students' strategies when solving an open-ended problem can be used to detect the differential effects of curricula that are more or less problem-based. This study is part of the large LieCal Project (CAI, 2014).

The LieCal Project was designed to characterize the differential effects of a problem-based curriculum, CMP, and more traditional middle-school mathematics curricula. Through this longitudinal study of curricular effect, we have found that CMP students show greater growth than their non-CMP counterparts on open-ended tasks that assess conceptual understanding and problem solving, while maintaining similar growth through middle school on computation and equation solving skills. Following these students into their high school years, we have found consistent differences between the former CMP and non-CMP students

that appear to reflect the different emphases of the CMP and non-CMP curricula. As a research finding, this continued curricular effect is particularly notable, as it passes beyond the grade band in which students encountered the curricula.

Fundamentally, it is important to assess students' mathematical learning using diverse tasks that reflect different aspects of that learning. In order to measure curricular effect more completely, one must attend to conceptual understanding and problem solving as well as procedural skill and fluency. In particular, for curricula that are designed to be problem based, it is critical to find ways to measure how students' problem-solving capacities develop over time. Here, we have shown that an analysis of problem-solving strategies using students' responses to an open-ended problem can indeed reflect differences in curricular effect, not only in the short term, but also longitudinally as students progress through their schooling.

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Appendix

Sally is having a party.

The first time the doorbell rings, 1 guest enters.
The second time the doorbell rings, 3 guests enter.
The third time the doorbell rings, 5 guests enter.
The fourth time the doorbell rings, 7 guests enter.

Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

A. How many guests will enter on the 10^{th} ring? Explain or show how you found your answer.

B. How many guests will enter on the 100^{th} ring? Explain or show how you found your answer.

C. 299 guests entered on one of the rings. What ring was it? Explain or show how you found your answer.

D. How many guests will enter on the n^{th} ring? Show or explain how you found your answer.

E. If we count all of the guests who entered on the first 100 rings, how many would we get in total? Show or explain how you found your answer.

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