

A Didactic Approach of Sociocultural Perspectives of Mathematical Modelling for Classrooms: An Ethnomodelling Perspective

Uma Abordagem Didática de Perspectivas Socioculturais da Modelagem Matemática para Salas de Aula: Uma Perspectiva de Etnomodelagem

Una Abordaje Didáctica desde las Perspectivas Socioculturales a la Modelación Matemática en las Salas de Clase:
Una Perspectiva desde la Etnomodelación

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ABSTRACT

Mathematical local ideas, procedures, techniques, and practices can be assessed by using different methodological approaches, such as the emic (local) and etic (global) approaches of the ethnomodelling process. Accordingly, we aimed to understand the association between the etic (global) and the emic (local) approaches through 'dialogic' because they are dynamic and complementary to each other as they can be taken as "two sides of the same coin." Thus, researchers and educators must develop investigations in the ethnomodelling field by using both approaches to better understand the mathematical knowledge developed by members from distinct cultural groups. Therefore, students need to be inserted in a learning environment that enables them to use mathematical knowledge previously acquired in school and tacitly developed and accumulated in their own communities. Thus, it is necessary to create a didactic approach of sociocultural perspectives of mathematical modeling for classrooms from an ethnomathematical and ethnomodelling viewpoint.

Keywords: Didactic Approach; Ethnomathematics; Ethnomodelling; Ethnomodels; Mathematical Modelling; Sociocultural Perspective.

RESUMO

As ideias, os procedimentos, as técnicas e as práticas matemáticas locais podem ser avaliadas utilizando diferentes abordagens metodológicas, como as abordagens êmica (local) e ética (global) no processo de etnomodelagem. Assim, buscamos entender a associação entre essas duas abordagens por meio da dialogia porque elas são dinâmicas e complementares entre si, podendo ser entendidas como *os dois lados da mesma moeda*. Assim, os pesquisadores e educadores devem desenvolver investigações no campo da etnomodelagem utilizando ambas as abordagens para entender melhor o conhecimento matemático desenvolvido por membros de grupos culturais distintos. Portanto, os alunos precisam ser inseridos em um ambiente de aprendizagem que os capacite a utilizar o conhecimento matemático previamente adquirido na escola e tacitamente desenvolvido e acumulado em suas próprias comunidades. Então, é necessário desenvolver uma abordagem didática para as perspectivas socioculturais de modelagem matemática para salas de aula a partir do ponto de vista da etnomatemática e da etnomodelagem.

Palavras-chave: Abordagem Didática; Etnomatemática; Etnomodelagem; Etnomodelos; Modelagem matemática; Perspectiva sociocultural.

RESUMEN

Las ideas, procedimientos, técnicas y prácticas matemáticas locales pueden evaluarse mediante el uso de diferentes enfoques metodológicos, como los enfoques emic (local) y etic (global) del proceso de etnomodelación. En consecuencia, nuestro objetivo fue comprender la asociación entre los enfoques etic (global) y emic (local) a través de la *dialogía*, ya que son dinámicos y complementarios entre sí, al ser dos lados de la misma moneda. Por lo anterior, los investigadores y educadores deben desarrollar investigaciones en el campo de la etnomodelación utilizando ambos enfoques para comprender mejor el conocimiento matemático desarrollado por miembros de distintos grupos culturales. Los estudiantes necesitan insertarse en un entorno de aprendizaje que les permita utilizar el conocimiento matemático previamente adquirido en la escuela y desarrollado y acumulado tácitamente

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en sus propias comunidades. En este sentido, es necesario crear un enfoque didáctico de perspectivas socioculturales de la modelación matemática para las aulas desde un punto de vista etnomatemático y de la etnomodelación.

Palabras clave: Abordaje Didáctico; Etnomatemáticas; Etnomodelación; Etnomodelos; Modelación Matemática, Perspectiva Sociocultural.

INITIAL CONSIDERATIONS

Mathematical modelling offers one of the most important research trends for the development of teaching and learning processes in mathematics (Rosa, 2000). In this regard, this particular pedagogical action can be directed towards the resolution of everyday problems, situations, and phenomena through the use of modelling to value and enable the connection between mathematics and the daily experiences lived by members of distinct cultural groups.

Mathematical modelling is the art of transforming reality problems into mathematical problems and solving them by interpreting their solutions in the language of the real world (Bassanezi, 2002). Modelling techniques provide the contextualization of academic and school mathematics by providing necessary conditions for the development of pedagogical actions through the elaboration of mathematical models, so that members of distinct cultures can act satisfactorily in the *glocalized world*³ (Rosa 2010).

In so doing, mathematical modelling presupposes the use of multidisciplinary approaches. This is because it makes use of diverse confluences and other trends in mathematics education, such as ethnomathematics, which is occupied with the removal of boundaries between various research areas (Bassanezi, 2002). Thus, mathematics education is a scientific field directed towards the teaching and learning process in mathematics through its resignification, which enables the understanding and perception of its importance in the development of pedagogical actions in classrooms (Rosa, 2010).

Similarly, ethnomodelling relates concepts of ethnomathematics with mathematical modelling (Rosa; Orey, 2010). For example, Caldeira (2007) states that it is necessary to consider mathematics constructed and signified in the cultural practices developed by the members of local communities. It is also necessary to consider the influences of these meanings in the didactic process in order to apply the assumptions of mathematical modelling as a means to achieve the objectives proposed for the conduction of its pedagogical action through its complementarity with ethnomathematics.

However, for this objective to be achieved, Rosa and Orey (2018a) argue that there is a need for students to be inserted in a learning environment that enables them to use mathematical knowledge that was previously acquired in the schools and also tacitly developed and accumulated in their own communities.

³ According to Rosa and Orey (2018b), a *glocalized world* enables the development of active, interactional, cooperative, collaborative, and dialogical processes that requires an ongoing negotiation between the local and the global mathematical, scientific, technological, and engineering knowledge through the development of a cultural dynamism.

ETHNOMATHEMATICS, MODELLING, AND ETHNOMODELLING

Historically, models that originate from within the reality of members of distinct cultural groups can be considered as pedagogical tools that are used for the abstraction of mathematical concepts, because these members develop their own set of ideas, techniques, and mathematical concepts, among which some basic tools are used in the development of the modelling process (Rosa; Orey, 2012a).

These tools can be understood as ways that members of each cultural group develop to deal with their own realities in order to mathematize and model their world through the use of measurement, comparison, quantification, classification, and inference developed locally (D'Ambrosio, 2017).

From the perspective of Cortes (2017), this context allows the exploration of ideas, procedures, and local mathematical practices, which aims to value and respect diverse cultural values and the knowledge acquired by students through their own experiences in society. Therefore, ethnomathematics and modelling are important tools to help students to understand, comprehend, analyze, and reflect on their own sociocultural context.

In this regard, being proficient in the use of modelling is of fundamental importance so that members of distinct cultural groups, through their actions, modify their own reality so that they can be included in the process of social transformation in a critical and reflective way (Rosa; Orey, 2015).

Through the modelling process, it is possible to show that a key aspect of this process is to help students realize their mathematical potential through the recognition of the importance of culture for the appreciation of their own identity because this aspect influences the way they think, learn, reflect, infer, and take informed decisions (Rosa; Orey, 2012a).

Thus, Rosa and Orey (2012a) state that mathematical modelling is a learning environment that facilitates the construction and transfer of mathematical knowledge through the use of their own mathematical knowledge which can be both: a) *explicit*⁴ and *tacit*⁵, which interact in this environment. In this context, members of distinct cultural groups have developed and are developing diverse and different ways of doing mathematics. Thus, D'Ambrosio (1990) states that members of these groups have developed, throughout history, distinct ways to mathematize their own reality by using elements of the modelling process.

Therefore, mathematization is the process through which members of distinct groups use different mathematical tools to help them organize, analyze, understand, understand, model and solve the problems faced in their daily lives (Rosa; Orey, 2006). These tools enable, according to Rosa and Orey (2017a), the identification of ideas in order to describe pro-

4 Explicit knowledge is related to a concrete fact, which can be disseminated by teachers through the use of textbooks, academic knowledge about the subject, knowledge of pedagogical instructional practices, and any other method of using materials and technological instruments that can help them to absorb, internalize and, consequently, transfer and diffuse the applicability of this knowledge to other areas of human knowledge (Rosa; Orey, 2012).

5 Tacit knowledge is the unwritten, unspoken, and hidden mathematical ideas, procedures, techniques, and practices developed by members of distinct cultural groups, which is based on their emotions, experiences, insights, intuition, observations, and internalized information developed through the resolution of phenomena they face in their daily life (Rosa; Orey, 2017a). It is integral to the consciousness of these members because it is acquired through association with members of other cultural groups and requires joint or shared activities to be imparted from one to another. It constitutes a set of informally developed knowledge and forms the underlying framework that makes explicit knowledge possible (Polanyi, 1966).

cedures and mathematical practices specific to a cultural context, which aim to help these members to discover relationships and regularities.

Hence, Rosa and Orey (2003) highlight that this cultural approach to modelling allows these members to schematize, formulate, and visualize problems and situations in different ways, which help them transcend the solution of real-world phenomena to mathematical conceptualization through the mathematization process

In this direction, Rosa and Orey (2017a) state that the sociocultural perspective of mathematical modelling involves the study of mathematical ideas, procedures, and practices found in diverse cultural contexts so that they can be used in their pedagogical actions in classrooms through the elaboration of ethnomodels.

Consequently, Rosa and Orey (2017b) argue that mathematical modelling procedures can be employed when ethnomathematics is actively used as a system based on a theoretical basis that helps members of distinct cultural groups in the development of, and the solving of everyday problems related to our diverse social, cultural, economic, political, and environmental contexts.

It is from this context that we see the connection between mathematical modelling and cultural aspects of mathematics through ethnomathematics, which is the study of ideas and procedures used in mathematical practices, which are developed by members of distinct cultures and considering the acquired mathematical knowledge and cultural practices in the community with the use of problem-solving techniques developed locally (Rosa; Orey, 2010).

These techniques are considered as the basic tools used by ethnomodelling that help teachers and researchers in carrying out the translation between the local (emic) and global (etic) approaches. Thus, ethnomodelling is a tool that aims to mediate the cultural forms of mathematics with the school curriculum to facilitate the development of the teaching and learning processes in mathematics.

According to this context, mathematical modelling is a teaching trend in mathematics education that aims to develop critical and reflective students who are aware of the different problems that are faced in their everyday life. However, for this objective to be achieved, there is a need for students to be given a learning environment that allows them to use of mathematical knowledge both acquired at school and tacitly in the community in which they are inserted. This approach will help students to contextualize curricular activities in the daily life of the students (Rosa; Orey, 2007).

Modelling techniques provide the contextualization of school/academic mathematics by providing the necessary conditions through the elaboration of mathematical models, so that members of distinct cultural groups can act in the globalized world. For example, this contextualization is an important concept for the development of citizens students, as it offers a great opportunity for the teaching of sociocritical efficiency⁶ (Rosa; Orey, 2017a).

⁶ The fundamental characteristic of sociocritical efficiency is the emphasis on students' critical analysis of society's power structures. Another important feature is the students' personal reflection on the social elements that underpin the globalized world. Thus, the critical perspective of students in relation to the social conditions that affect their own experiences can help them to identify common problems and, collectively, develop different strategies to solve them (Rosa; Orey, 2007).

In this regard, educators have the responsibility to favor the establishment of relationships between school/academic mathematics and students' tacit knowledge, so that learners can perceive the presence of mathematics in the activities they experience daily. Thus, it is necessary that, in the classrooms, teachers discard the traditional transmissive pedagogical model and favor the transformative pedagogical model (Rosa; Orey, 2006).

Many traditional teaching methods predominant in many educational systems tend to focus on the traditional objective of learning for the transmission of mathematical knowledge (Rosa, 2010). In this context, modelling as a learning environment in which teachers and students become collectively responsible for the development of mathematical knowledge and for the conversion between its tacit and explicit dimensions, from situations arising, preferably, from their own realities (Rosa; Orey, 2012a).

In this perspective, the conception of the role of students in this approach is that all participants become active collaborators in the learning process, which is a more stimulating task than the one related to the simple reception of mathematical knowledge and practices.

Hence, in ethnomodelling, a form of sociocritical mathematical modelling, students can be considered as creators of mathematical knowledge, as this process provides the conditions for them to get involved with mathematics, so that they can challenge it, understand it, and interpret it by making it into a product of human creation.

In this direction, Cortes (2017) states that learning is triggered according to the students' purpose, as it develops differentiated capabilities so that they can act, react, reflect, and change the environment in which they live by transforming it, strategically. Thus, these environmental influences the development of students' cognitive process in different ways, as it is related to their sociocultural context.

Therefore, the sociocritical dimension of mathematical modelling has as a background sociocultural and social knowledge theories, which are related to the emancipatory perspective and transformative learning that applies philosophical ideals of critical thinking theory (Rosa; Orey, 2007). Sociocultural theories are related to learning processes that are triggered through how learners are socialized, as knowledge is better constructed when students interact to socializing learning.

Thus, students act both cooperatively and collaboratively in order to support and encourage each other, so that they can reflect on the solution of problems rooted in authentic situations. As in the mathematical modelling process, students must actively participate in the construction of their mathematical knowledge by connecting it interdisciplinary with other knowledge areas in an interdisciplinary fashion (Rosa; Orey, 2017a).

In the sociocultural theory, Rosa and Orey (2007) affirm that the joint work between teachers and students makes learning more effective, because cultural tools, such as artifacts, language, traditions, behaviors and institutions are shared. So it is that learning is constructed in a social context, as members of different cultural groups learn collaboratively through integrated experiences.

Similarly, Habermas (1971) proposed a theory of social knowledge, which is an important contribution to the development of a theoretical basis for critical thinking. According to this theory, the effects of social structure on knowledge influence the many diverse ways of knowing that are acquired by individuals in equally diverse social environments. In this case, knowledge that members of distinct cultures produce are partially determined by the interests that stimulate and motivate them.

In this context, Habermas (1971) suggests that there are three areas of knowledge: a) technical or work, b) practical or interaction, and c) emancipatory or powerful, which influences three aspects of the social existence: a) technical knowledge or work, b) practical knowledge or interaction, and c) emancipatory knowledge or power. These three areas of knowledge are exemplified as follows:

- a) *Technical knowledge or work* is defined by the way in which members of distinct cultural groups control and manipulate their own sociocultural environment. It is the knowledge acquired through empirical and analytical investigations and also through technical rules. The focus of this knowledge is prediction and control. In this model, you are looking for:
 - Knowing what the observable attributes of a specific phenomenon are.
 - Knowing whether a specific result can be produced and reproduced.
 - Determine the correct answers.
 - Apply and use efficient rules to select different ways to manipulate the environment (Brown, 1984). In this knowledge, exercises are prescribed for students with the aim of directing them to obtain the correct answer.
- b) *Practical knowledge or interaction* is acquired through social communication and interaction. Members of distinct cultural groups communicate with one another in the form of symbolic interaction by using hermeneutics and interpretive discipline. In this model, it is sought to know:
 - Whether social actions are modified through communication.
 - What is the true meaning of communication.
 - It is in this type of knowledge that the meanings and interpretation of communicative patterns interact to build and elaborate the community understanding that serves to outline the normative agreement for social action.
- c) *Emancipatory knowledge or power* is defined by the acquisition of insights through self-empowerment that seek to emancipate members of distinct cultural groups from the institutional forces that limit and control the lives of these members. In this model, it is sought to know:
 - What are the social conditions that can cause misunderstanding of communication?
 - What tactics can be used to release a particular oppressive and repressive force?
 - What risks are involved in these tactics?

In the perspective of Rosa and Orey (2007), this type of knowledge aims to emancipate members of distinct cultures from the different modes of social domination. Corrobo-

rating with this perspective, according to Barbosa (2006), it is necessary for students to develop their ability to solve problems, decisions, work in teams, and communicate effectively, which are important characteristics of sociocritical mathematical modelling.

Consequently, Kaiser and Sriraman (2006) and Rosa and Orey (2007) argue about the need to apply the notions of the emancipatory approach to mathematical modelling, which educational objectives address issues of a sociopolitical nature and their consequences in the pedagogical practices used in schools. For example, this emancipatory approach is based on the sociocritical competence of members of distinct cultural groups in which its main objective is to help students to face and solve challenges imposed by the globalized society.

According to Rosa and Orey (2007), this approach must be directed to transform students into flexible, adaptable, reflective, critical, and creative citizens through the use of alternative pedagogical methodologies that aim to value and record ideas, procedures, and mathematical practices that are developed in distinct cultural contexts. Thus, the sociocritical aspects of modelling are based on the expansion of students' autonomy, as it aims to provide a critical reading of their worldview, as well as the development of an autonomous thinking, whose objectives are to contribute to the full exercise of their citizenship.

EMIC (LOCAL), ETIC (GLOBAL), AND DIALOGICAL (GLOCAL) ETHNOMODELS

Culture is a lens that shapes reality, as well it provides us with a blueprint that specifies a plan of action or expectations. At the same time, there are aspects of a culture that are unique to the members of distinct cultural groups, who together have grown, learned, and acted daily in diverse contexts, such as economic, social, cultural, political, and environmental in which they live. Hence, ethnomodelling is linked to mathematical practices developed by members of distinct cultural groups that tend to be of benefit by the presentation and organization of mathematical ideas and procedures that enable a development of communication, diffusion, and further transmission through generations (emic knowledge) (Rosa; Orey, 2015).

The representative idea of this approach gives room to the development of mathematical knowledge through scientific methods that help researchers and educators to build and understand the world (etic knowledge) by using small units of information named *ethnomodels* that make these representations (Cortes, 2017).

In this regard, Rosa and Orey (2012a) state that ethnomodels are considered as cultural constructs because one of the main objectives of its elaboration is to comprehend the way of thinking of these members, as well as to understand how they organize and model their own mathematical ideas, techniques, and procedures from their own point of view in order to mathematize their own reality. On the other hand, Eglash, Bennett, O'Donnell, Jennings, and Cintorino (2006) state that a model built without a first-hand sense of the world being modeled should be viewed with suspicion.

Researchers and educators, if not hindered by their prior ideology, paradigms, cosmologies, and worldviews, should create a sense of informed sense of the distinctions that

are effective from the point of view of the mathematical knowledge of the world being modeled. In so doing, they should be able to inform the outsiders (etic/global) what matters to the insiders (emic/local) (Rosa; Orey, 2017a).

Ethnomodelling emphasizes the organization of mathematical ideas, techniques, and procedures developed by the members of distinct cultural groups in order to facilitate its communication and transmission across generations, which adds cultural aspects to the modelling process (Rosa; Orey, 2017b). In this regard, these members construct ethnomodels of their own mathematical practices found in the sociocultural system they are in, and which links cultural heritage with the development of mathematical practices (Cortes, 2017).

This approach helps in the organization of pedagogical action in classrooms by using emic (local) and etic (global) aspects of mathematical knowledge through the development and elaboration of ethnomodels, which are described as cultural artifacts that are pedagogical tools used to enable the understanding of systems present in the reality of the members of distinct cultural groups (Cortes; Orey, 2020).

In this regard, ethnomodels are considered external representations that are both precise and consistent with scientific and mathematical knowledge that is socially constructed, developed, and shared by members of distinct cultures. The main objective in the elaboration of ethnomodels is to translate local (emic) constructs such as mathematical ideas, procedures, techniques, and practices in order to establish relations between local conceptual knowledge and the mathematics embedded in these constructs through dialogical relations (Eglash et al, 2006).

Ethnomodels link the development of mathematical practices to the cultural heritage of the members of distinct cultural groups, who possess necessary information to solve problems and situations described in systems in their own reality (Rosa; Orey, 2018b). The emphasis of ethnomodelling considers the processes that help in the construction and development of local mathematical knowledge systems, and which include collectivity, creativity, and inventiveness (Ascher, 2002) through the elaboration of ethnomodels.

According to this approach, it is impossible to imprison mathematical ideas, procedures, and practices in registers of univocal designation of reality because there are distinct systems that provide unambiguous representations of reality as well as universal explanations. This means that mathematics cannot necessarily be conceived as a universal language because its principles are not always the same everywhere around the world. In this context, the production process of mathematical ideas, procedures, and practices operates within the register of interpretative singularities regarding possibilities for symbolic construction of local mathematical knowledge (Rosa; Orey, 2015).

As stated earlier, ethnomodelling studies local mathematical processes developed by members of distinct sociocultural groups. Many interesting ethnomodels have been formulated by using data obtained from studies related to ethnomathematics, and which propose a rediscovery of knowledge systems adopted by the members of diverse groups (Basanezi, 2002, Babbitt, Liles; Eglash, 2012; Rosa; Orey, 2015). When this knowledge applies to mathematical ideas, techniques, and procedures through the elaboration of ethnomodels,

members of distinct cultural groups are able to understand the origin of mathematical practices more efficiently.

In ethnomodelling research, emic (local) constructs represent the accounts, descriptions, and analyses of mathematical ideas, techniques, procedures, and practices expressed in terms of conceptual schemes and categories that are regarded as meaningful and appropriate by members of the cultural group under study (Lett, 1996).

This means that emic (local) constructs are in accordance with the perceptions and understandings deemed appropriate by the insider's culture. The validation of these constructs comes with a matter of consensus from those who do the mathematics under study, in which local people who agree that emic constructs match shared perceptions, behaviors, and knowledge that portray characteristics of their own culture (Lett, 1996).

Emic (local) mathematical knowledge can be obtained through elicitation and observation because observers infer local perceptions. In emic (local) approaches, researchers and educators must put aside their own bias, prior theories, and assumptions in order to let those who develop the mathematical activity under study to explain their own practices (Cortes; Orey, 2020).

This approach allows for understanding mathematical themes, patterns, and concepts that emerge locally. Some of its strength lies in its appreciation of the uniqueness of the context being studied in its respect for local viewpoints, and potential to uncover unexpected mathematical findings (Rosa; Orey, 2018a).

On the other hand, "etic constructs are accounts, descriptions, and analyses of mathematical ideas, procedures, and practices expressed in terms of conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers" (Lett, 1990, p. 130). An etic approach uses these concepts as a starting point for innovative and new theories, hypothesis, perspectives, and concepts from outside of the cultural setting being studied.

Etic constructs provide us with independent, precise, logical, comprehensive, replicable, and observers (Rosa; Orey, 2017b). The validation of etic knowledge becomes a matter of logical and empirical analysis. In particular, the logical analysis of whether the construct meets the standards of comprehensiveness and logical consistency of concepts (Lett, 1990). It is important to emphasize that this particular research technique that is used in the acquisition of scientific and mathematical knowledge has no bearing on the nature of that knowledge.

Etic (global) knowledge may be obtained at times through elicitation, as well as observation. One of the strengths of the etic approach is that it allows for comparison across contexts and populations, and the development of more general cross-cultural concepts (Morris, Leung, Ames; Lickel, 1999).

On the other hand, etic ethnomodels are mathematical representations elaborated through a descriptive and external observations (Rosa, Orey, 2012a). Thus, the representation of local mathematical knowledge is developed when the members of different cultural

groups have their own interpretation of their culture (emic approach) as opposed to the researchers' interpretation and investigators who develop representations of mathematical knowledge place from the perspective of their own conceptions (etic approach).

In academic mathematics, learning constantly works with the elaboration and interpretation of mathematical models, which are considered as a "set of symbols and mathematical relationships that they somehow represent the studied object" (Bassanezi, 2002, p. 20). In this regard, if mathematics can be considered as a cultural construct, then it is equally a product of cultural development (Rosa; Orey, 2017b).

This assertion contradicts the claims that modern mathematics is universal, objective, and culturally neutral. If mathematical knowledge results from sociocultural interactions in which relevant ideas, facts, concepts, principles, and skills are acquired as a result of the influence of cultural contexts, then mathematics is not a universal formal domain of knowledge (Dossey, 1992).

On the other hand, there are at the very least six universal mathematics activities: counting, measuring, designing, locating, explaining, and playing (gambling, guessing, and explaining) that are thought to be practiced by the members of distinct cultural groups. These activities are also widely developed across cultures in order to provide the fundamental facets used to probe traditional daily living and scientific and mathematical practices, which are inseparably intertwined with other aspects and activities of any culture (Bishop, 1991).

It is important to highlight that many ethnomodels are often developed by considering the relationship with an ethnomathematics perspective. In this context, ethnomodels can be defined as intersections of emic, etic, and dialogical.

Emic (Local) Ethnomodels

Emic (local) ethnomodels can be considered as representations that are developed by members of different cultural groups themselves. They are based on mathematical concepts rooted in cultural aspects. Emic focuses on studying the sociocultural features of mathematical phenomena from within a specific cultural context in order to understand it as their members comprehend it (Rosa; Orey, 2012a).

In keeping with the emic perspective, a demand for local or culturally specific approaches has emerged besides the Western and Euro-American approaches. Examples of local or culturally specific topics and research instruments encouraged some researchers to support the valorization of emic perspective. Many theories and methods seem to be susceptible to cultural differences, as well as to the demand of cultural contextualization into the mathematics curriculum in schools (Rosa; Orey, 2010).

For example, Rosa and Orey (2015) affirm that emic ethnomodels are not intended to compare observed mathematical patterns in distinct contexts because comparisons are not evaluative. Emic constructs are the ones that apply only to specific cultural contexts in which mathematical ideas, procedures, and practices are developed overtime.

Usually, according to Rosa and Orey (2015), emic ethnomodels focus on a single culture and employ descriptive and qualitative methods to study mathematical ideas, proce-

dures, and practices. They focus on the study within a given cultural group context in which researchers and educators develop research criteria relative to internal characteristics or logic features of cultural systems. In this regard, meaning is gained relative to the context and therefore not transferable to other contextual settings.

Etic (Global) Ethnomodels

Etic (global) ethnomodels refer to mathematical phenomena that are constant or common throughout the world, and which do not vary all that much across cultures. Some mathematical concepts are generalized across cultural groups and the general idea of mathematical practices is considered a universal phenomenon (Kline, 1953). In this perspective, an etic approach understands the mathematical phenomena across cultures rather than culturally specific meanings (Rosa; Orey, 2012b).

Therefore, etic ethnomodels are more comparative because they examine different cultural mathematical practices by using standardized methods. These ethnomodels develop mathematical-theoretical ideas that are assumed to be applied in all cultural groups (Rosa; Orey, 2018b). However, it is naive to state that members of distinct cultural groups do not share universal mathematics characteristics.

For example, Bishop (1991) states that many of the everyday activities of members of cultural groups involves a substantial amount of mathematical application. In this context, Rosa and Orey (2017) state that etic ethnomodels the observations and data that are constructed according to a researchers' system of categories, metrics, and definition.

Dialogical (Glocal) Ethnomodels

Dialogical (glocal) ethnomodels use both emic (local) and etic (global) knowledge in order to understand processes of dialogue and dynamics of the encounters between cultures. While the traditional concepts of emic and etic perspectives provide important points of view for understanding and comprehending cultural influences on ethnomathematics and mathematical modelling research, this third perspective is important for the development of ethnomodelling investigations.

Hence, etic claims state that the knowledge of members of any given cultural group has no necessary priority over its competing emic claims. This is because there is a relation of interdependence between these two approaches. This perspective stresses the development of investigations through a mutual exchange between the emic and etic approaches, which could be considered as a qualitative transformation in the research process (Rosa; Orey, 2018b).

The rationale behind the emic-etic (dialogical) approach is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur. The emic (local) approach tries to investigate the mathematical phenomena and their interrelationships and structures through the eyes of the native people to a particular cultural group. Thus, in dialogical approaches, the etic perspective claims acknowledge of the given cultural group has no necessary priority over its competing emic claims.

In this point of view, Eglash et al. (2006) stated that there is a necessity to depend “on acts of translation between emic and etic perspectives” (p. 347). In this regard, cultural specificity may be better understood with the background of communality and the universality of theories and methods and vice versa. In this regard, it is necessary that the insights that have been acquired through subjective and culturally contextualized methods be verified with methods independent of the subjectivity of the observer and researcher in order to achieve a scientific character.

According to this perspective, Rosa and Orey (2007) argue that there are some cultural differences found within these six universal mathematical activities. Even though they may be considered universal, it is important to recognize that they are merely universal to those members who share the same cultural features, historical perspectives, and linguistic backgrounds. This means that school/academic mathematics may look the same in many cultures because there is a competitive social, economic, environmental, and political ethics that demands distinct mathematical developments.

According to Cortes (2017), it is in this unique context and assemblage of culturally constructed symbolisms that enables the manipulation of the representations of mathematical knowledge because members of distinct cultures develop procedures in their cognitive systems, which presents a process that occurs in the context of socially constructed activities.

Mathematical skills that students learn in schools are not logically constructed based on abstract cognitive structures, but rather forged out of a combination of previously acquired mathematical tacit knowledge and innovative cultural inputs (Rosa. Orey, 2018a). Therefore, mathematics arose out of the needs of organized communities, which cannot be divorced from the activities and practices developed locally and globally in a *glocalized society*⁷.

A DIDACTIC EXAMPLE OF ETHNOMODELLING: MATHEMATIZING SONA

Since ethnomodelling proposes a dialogue between local (emic) and school and academic approaches (global) to the construction of mathematical knowledge through cultural dynamism. The development of this process increases the potential for continual growth in the debate related to the nature of mathematical modelling and how it links to culture. It offers a basis for decision making processes in the elaboration of ethnomodels by analyzing the mathematization procedures in the production of cultural artifacts (Rosa; Orey 2017b).

Consequently, D'Ambrosio (1993) discusses how cultural artifacts provide necessary material tools that help in the development of clothing, shelters, navigation, defense, and transportation, which have come to assist members of distinct cultural groups to solve daily

⁷ The neologism glocalization is composed of the terms globalization and localization, which has emerged as the new standard in reinforcing positive aspects of worldwide interaction, be it in textual translations, localized marketing communication, knowledge acquisition, and sociopolitical considerations. The concept of o a glocalized society follows a sociological/historical approach regarding societies in their dynamic social transformations (Rosa; Orey, 2017a).

problems by using their own mathematical techniques and strategies. These artifacts are considered as tools, devices, and *instruments of observation*.

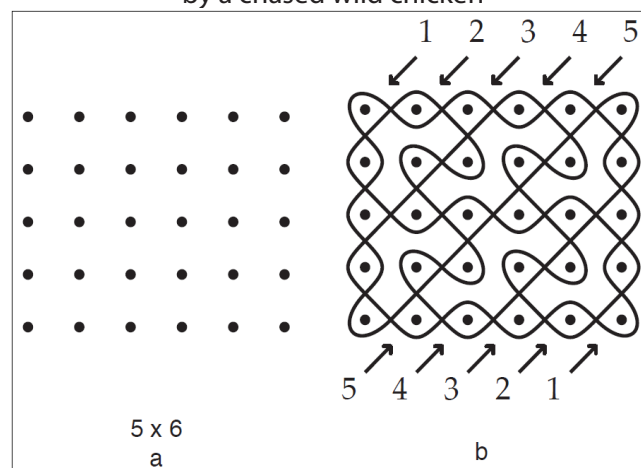
For example, the results of work conducted by Gerdes (2014) helped in developing the comprehension and the mathematization of sand drawings called *sona* that are created by the *Côkwe* people who live in Angola, Democratic Republic of Congo, and Zambia. These *sona* cultural artifacts help to legitimize and value this cultural practice by translating this knowledge between distinct mathematical systems such as academic mathematics.

When the *Côkwe* gather in the center of their villages or hunting camps, they usually sit around a fire or in the shade of trees in order to spend their time in conversations illustrated on the ground by *sona* drawings originating in local traditions that include proverbs, fables, games, riddles, and animals. It plays an important role in imparting knowledge and wisdom from generation to generation (Gerdes, 2014).

The drawings must be done smoothly and continuously because any hesitation or interruption is interpreted by the members of this particular cultural group as imperfections or lack of knowledge. Thus, in order to make it easier for them to learn and develop their own pictograms, design specialists have invented an interesting mnemonic feature: after cleaning and smoothing the ground, these members begin by marking with their fingertips an orthogonal network of equidistant points, and the number of rows and columns depends on the representation of an observed phenomenon.

For example, Gerdes (2014) states that in order to represent the footprint left by a chased chicken by elaborating a glocal ethnomodel that requires five rows and six columns (figure 1). An analysis of the symbolic representation of this route shows its mathematization: $5 \times 6 = (1+2+3+4+5) + (5+4+3+2+1) = 2(1+2+3+4+5)$.

Figure 1 - Dialogic (Glocal) ethnomodel that represents the route described by a chased wild chicken

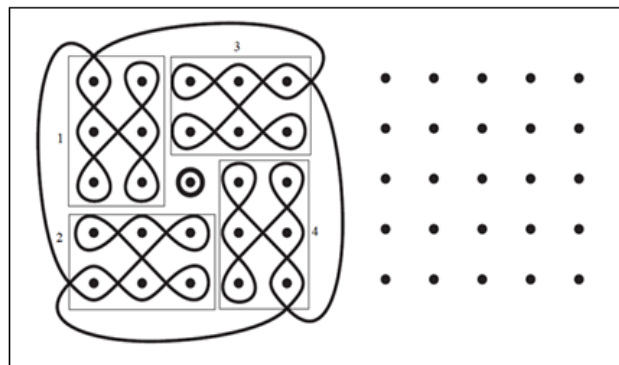


Source: Adapted from Gerdes (2014, p. 22)

This glocal ethnomodel can be used as a starting point to the study of arithmetic progressions by elaborating the following ethnomodel: $2(1 + 2 + 3 + \dots + n) = n(n + 1)$ or alternatively $2(1+2+3+ \dots + n) = (n+1)^2 - (n+1)$ that can be used as $2(1+2+3+4+5) = 6^2 - 6$.

According to Gerdes (2014), each *Côkwe* design redistributes the points of a referential network of the drawings. For example, figure 2 shows a representation of a forest with some *qundu* birds that may corresponds to an arithmetic relation.

Figure 2 - Representation of a forest with some *qundu* birds



Source: Adapted from Gerdes (2014, p. 20)

Based on this representation, figure 3 shows the following arithmetic relation is found: $5^2 = 4 \times (3 \times 2) + 1 = 25 = 4 \times 6 + 1 = 25 = 24 + 1 = 25 = 25$.

Figure 3: Sona representation of the arithmetic relation $52 = 4 \times (3 \times 2) + 1$

Source: Adapted from Gerdes (2014, p. 20)

What makes this issue interesting is how the *Côkwe* had not studied academic mathematics, yet their sand drawings incorporate patterns that have significant connections with diverse features of school mathematics. This is one concrete example of how it is possible to apply local mathematical ideas in the context of teaching mathematical content.

It is important, as well as enjoyable to seek the construction of effective connections between *sona* geometry and schools' mathematics. This context is why we can state that this approach can be used as a pedagogical action in the mathematics classrooms in order to help students to (re)discover arithmetic relationships by developing ethnomodels.

Thus, Rosa and Orey (2012b) state that the sociocultural aspects of mathematical practices used in classrooms are related to the mathematical content and processes that associate them with pedagogical approaches used for the processes of teaching and learning mathematics. From this perspective, the acquisition of mathematical knowledge is influenced according to the sociocultural considerations of the members of distinct cultures who participate in the school communities.

FINAL CONSIDERATIONS

Local ideas, procedures, techniques, and practices can be assessed by using different methodological approaches, such as the emic and etic approaches in the ethnomodelling process. Accordingly, we aimed at understanding the association between the etic (global) and the emic (local) approaches through *dialogicity*, because they are dynamic and complementary to each other—they can be taken as “two sides of the same coin”.

Results in the studies by Cortes (2017) show that ethnomodelling allowed an integrating approach in mathematics' school discipline matrix, because it took into consideration both the emic and etic mathematical knowledge. Therefore, professors and students could holistically and broadly understand mathematical information developed by members from different cultural groups composing the school students' population.

Thus, the dialogical principle is important for the conduction of ethnomodelling investigations, since it develops the sense of wholeness (global, etic); therefore, it cannot only be taken as overlapping of locations (local, emic). Therefore, based on such a principle, neither "globality" nor "locality" prevails over one another, since there is a dialogue that must take place between these two approaches.

The ethnomodelling process demands the promotion of dialogue (dialogical approach) between the local mathematical practices (emic approach) and those of the academia (etic approach), since it would enable the approximation of these two knowledge fields through mathematical and contextualizing means. Thus, researchers must develop investigations in the ethnomodelling field by using both approaches, so they can better understand the mathematical knowledge developed by members from different cultural groups.

From the dialogical viewpoint, the conduction of investigations methodologically supported by these two approaches (local and global) makes it possible to reach the full and broad understanding of mathematical knowledge developed by members from different cultural groups. Finally, one of the main goals of conducting investigations in the ethnomodelling field lies on the acquisition of emic and etic knowledge through dialogues set between these two approaches.

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